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Phil 184/284: Formal and Informal Epistemology
Section 4 Handout

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Definitions:

- $Con(S) = \{logical\ consequences\ of\ S\}$
    1. Example:
        - Consider the set $\{p,\ p \rightarrow q\}$
        - The set entails the propositions q and $\neg p \vee p$ (among others)
            - (Informally) A set entails some proposition φ just in case φ is true every time the propositions in the set are true
        - Then, $q, \neg p \vee p \in Con(\{p,\ p \rightarrow q\})$
- Expansion =
    1. This is (generally) going from suspension of judgment about *P* to believing *P*
    2. Definition:
        - $(+)(\boldsymbol{B} + P) = \{\varphi : \{\boldsymbol{B} \cup \{P\}\} \Rightarrow \varphi\}$
        - In English:
            - $\boldsymbol{B}$ expanded by *P*—i.e. $(\boldsymbol{B} + P)$—is the set of all sentences φ such that φ is entailed by the union of $\boldsymbol{B}$ and *P* (where the union of two sets is a set containing every member that is in either (or both) of the two sets)
    3. Example:
        - Let $\boldsymbol{B} = con(\{A \rightarrow B\})$
        - Then, $(\boldsymbol{B} + A) = \{\varphi : \{\boldsymbol{B} \cup \{A\}\} \Rightarrow \varphi\} = \{A \rightarrow B, A, B, \neg A \vee A, \ldots\}$
- Maximal non-P belief subset $\boldsymbol{B}^*$ of set $\boldsymbol{B}$ =
    1. Definition:
        i. $\boldsymbol{B}^* \subseteq \boldsymbol{B}$
            1. (every member of $\boldsymbol{B}^*$ is a member of B)
        ii. $P \notin \boldsymbol{B}^*$
            1. (P is not in $\boldsymbol{B}^*$)
        iii. $\neg(\exists \boldsymbol{B}^{\$})[(\boldsymbol{B}^* \subset \boldsymbol{B}^{\$})\ \&\ (\boldsymbol{B}^{\$} \subseteq \boldsymbol{B})\ \&\ (P \notin \boldsymbol{B}^{\$})]$
            1. (there is no other set $\boldsymbol{B}^{\$}$ where $\boldsymbol{B}^*$ is a smaller subset of $\boldsymbol{B}^{\$}$**,** where $\boldsymbol{B}^{\$}$ is a subset of $\boldsymbol{B}$ and where *P* is not in $\boldsymbol{B}^{\$}$)
    2. Example:
        - $\boldsymbol{B} = con(\{P, Q\})$
        - You then lose your belief in Q
        - $\boldsymbol{B}^* = con(\{P\})$
            - $\boldsymbol{B}^* \subseteq \boldsymbol{B}$: Every member of *B* is in *B* (since any consequence of *P* is a consequence of *P* and *Q*
            - $B \notin \boldsymbol{B}^*$: *P* is not in the set $\boldsymbol{B}^*$
            - $\neg(\exists \boldsymbol{B}^{\$})[(\boldsymbol{B}^* \subset \boldsymbol{B}^{\$})\ \&\ (\boldsymbol{B}^{\$} \subseteq \boldsymbol{B})\ \&\ (P \notin \boldsymbol{B}^{\$})]$: there is no other set $\boldsymbol{B}^{\$}$ which
                - 1) is a bigger than $\boldsymbol{B}^*$,
                - 2) while $\boldsymbol{B}^{\$}$ is still a subset of $\boldsymbol{B}$
                - 3) and *P* is not in $\boldsymbol{B}^{\$}$
- What if there are *two* or more maximal subsets?
    1. Example:

        - $\boldsymbol{B} = con(\{P, Q, R\})$
        - You then learn $\neg(P\&Q)$, so you have to first suspend judgment about $(P\&Q)$
            - Note: contraction is (generally) suspending judgments about prior beliefs, not disbelieving something you previously believed, but it is a step *toward* disbelief/revision
        - Then, two maximal subsets $\boldsymbol{B}^*$ are
            - $\boldsymbol{B}^{\mathbf{1}*} = con(\{P, R\})$
            - $\boldsymbol{B}^{\mathbf{2}*} = con(\{Q, R\})$
        - What should your belief set be?
- Entrenchment = how *useful* a sentence is in inquiry and deliberation[1]
- Contraction = $(\boldsymbol{B} - P) =$
    1. This is (generally) going from belief in *P* to suspending judgment about *P*
    2. Definition:
        - If there are some sets $\boldsymbol{B}^*$**s** which are maximal subsets of $\boldsymbol{B}$ and no other maximal subset is more entrenched,
            - then $(\boldsymbol{B} - P) = \cap\, \boldsymbol{B}^*$**'s** (i.e. the contraction of $\boldsymbol{B}$ by *P* is the set of all elements that are *shared by all* the most entrenched maximal subsets)
        - If there are no such sets,
            - then $(\boldsymbol{B} - P) = \boldsymbol{B}$
    3. Examples:
        - Example 1:
            - Recall:
                - $\boldsymbol{B} = con(\{P, Q, R\})$
                - $\boldsymbol{B}^{\mathbf{1}*} = con(\{P, R\})$
                - $\boldsymbol{B}^{\mathbf{2}*} = con(\{Q, R\})$
            - What is $(\boldsymbol{B} - (P\&Q))$?
                - If $\boldsymbol{B}^{\mathbf{1}*}$ is more entrenched than $\boldsymbol{B}^{\mathbf{2}*}$, then $(\boldsymbol{B} - (P\&Q)) = \boldsymbol{B}^{\mathbf{1}*} = con(\{P, R\})$
                - If $\boldsymbol{B}^{\mathbf{2}*}$ is more entrenched than $\boldsymbol{B}^{\mathbf{1}*}$, then $(\boldsymbol{B} - (P\&Q)) = \boldsymbol{B}^{\mathbf{2}*} = con(\{Q, R\})$
                - If $\boldsymbol{B}^{\mathbf{1}*}$ and $\boldsymbol{B}^{\mathbf{2}*}$ are *equally* entrenched, $(\boldsymbol{B} - (P\&Q)) = \boldsymbol{B}^{\mathbf{1}*} \cap \boldsymbol{B}^{\mathbf{2}*} = con(\{R\})$
        - Example 2:
            - $\boldsymbol{B} = con(\{P, Q, R\})$
            - What is $(\boldsymbol{B} - (P \vee \neg P))$?
                - There is no maximal subset (qua belief set) which does not contain $(P \vee \neg P)$, so $\left(\boldsymbol{B} - (P \vee \neg P)\right) = \boldsymbol{B}$
- Revision =
    1. This is going from belief in *P* to believing not *P*
    2. $(\boldsymbol{B} * P) = ((\boldsymbol{B} - \neg P) + P)$

[1] "The fundamental criterion for determining the epistemic entrenchment of a sentence is how useful it is in inquiry and deliberation. Certain pieces of [information] are more important than others when planning future actions, conducting scientific investigations, or reasoning in general . . . The epistemic entrenchment of a sentence is tied to its explanatory power and its overall informational value within the belief set."

1) Let $\boldsymbol{B} = Con(\{p, p\&r\})$. Furthermore, let every maximal subset of $\boldsymbol{B}$ containing q be more entrenched than any maximal subset of $\boldsymbol{B}$ not containing q, and let every other maximal subset of $\boldsymbol{B}$ be equally entrenched. State whether each of the following claims is true or false, and explain your answer

   a) $r \in \boldsymbol{B} + q$

This is true. $\boldsymbol{B}$ expanded by q—i.e. $(\boldsymbol{B} + q)$—is the set of all sentences φ such that φ is entailed by the union of $\boldsymbol{B}$ and q. r is entailed by $\boldsymbol{B}$ since $r \in Con(\{p, p\&r\})$, so it must also be entailed by $\boldsymbol{B}$ in union with any other set. Hence, $q \in \boldsymbol{B} + Y$.

2) Present two of your own counter-examples to AGM theory. For each example, explain which postulate or postulates it threatens.
   a. **USE YOUR OWN WORDS AND EXAMPLES!!**

- Logical omniscience objection:
  1. Expansion postulate:
     - (+1) $(\boldsymbol{B} + P)$ is fully logical (it is closed under implication—i.e. if you believe a set of propositions, you also believe everything entailed by those propositions)
       - Recall $(+)(\boldsymbol{B} + P) = \{\varphi : \{\boldsymbol{B} \cup \{P\}\} \Rightarrow \varphi\}$
  2. Objection:
     - A fully rational agent does not always believe the consequences of their beliefs
     - For example:
       - A mathematician might believe in basic mathematical facts which entail the Riemann hypothesis despite not believing the hypothesis
     - This kind of objection raises questions about what the intended target domain of the model is (e.g. an ideally rational vs. somewhat actual rational agent)
- Monotonicity objection:
  1. Expansion postulate:
     - (+5) If everything in $\boldsymbol{B}$ is also in $\boldsymbol{B}^*$, then everything in $(\boldsymbol{B} + P)$ is also in $(\boldsymbol{B}^* + P)$
  2. Objection:
     - Rational belief is non-monotonic (i.e. learning more information can cause you to retract a belief or inhibit the acquisition of a belief)
     - This paves the way to a counter-example for (+5):
       - Propositions:
         - A =99% of Canadians speak English and not French
         - B = Montreal is in Canada
         - C = 50% of Montreal Canadians speak French and not English
         - P = Pierre is a Montreal Canadian
         - E = Pierre speaks English and not French
       - Belief sets:
         - $\boldsymbol{B} = con(\{A, B\})$
         - $\boldsymbol{B}^* = con(\{A, B, C\})$
       - Then:
         - $E \in \boldsymbol{B} + P$ and $\boldsymbol{B}^* \subseteq \boldsymbol{B}$, but $E \notin \boldsymbol{B}^* + P$
         - In other words, everything in $\boldsymbol{B}$ is also in $\boldsymbol{B}^*$, but everything in $(\boldsymbol{B} + P)$ is *not* in $(\boldsymbol{B}^* + P)$ because additional information in $\boldsymbol{B}^*$ stops one from inferring E