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Phil 184/284: Formal and Informal Epistemology
Section 2 Handout

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Notes on Chapter 2 of Rational Mind:

- Two ways a model of mind might fail:
    1. Inaccuracy in what it does contain
    2. Omission of what it should contain
- Terminology:
    - Credence = confidence
    - Unit interval = $[0,1]$ = the set of numbers containing all numbers between 1 and 0 and the numbers 1 and 0 themselves
- Bayesian model:
    1. Credences are modelled as probabilities
    2. These obey specific synchronic and diachronic norms of rationality
- Features of the Bayesian model:
    1. There are uncountably many types of credence
        - 'uncountable' infinity = any set such that the number of its members cannot be paired-up without remainder with the counting numbers (i.e. the natural numbers).
    2. They are linearly ordered by strength
    3. They are infinitely precise in intrinsic detail
- Further features of the Bayesian model:
    - Target domain is an *idealized*, fully rational agent
    - Subjective probability: "concerns our perspectival take on the world"
- Synchronic norm: Credence partition principle CrPP
    - When $\{p_1, \ldots, p_n\}$ are a logical partition: $cr(p_1) + \cdots + cr(p_n) = 100\%$
    - Conditional credence rule:
        - (cr/) $cr(C/A) =_{df.} cr(A\&C)/cr(A)$, when $cr(A)$ is non-zero.
- Diachronic norms: Bayesian transition rules
    - (J-Cond) When your original credence distribution lends A in-between credence, but new input rationally shifts your view to a new in-between credence, then, in those circumstances, your new take on any claim C should line up with your new take on A together with your old take on C given A/¬A. Specifically: $cr_{new}(C)=\{[cr_{new}(A)\times cr_{old}(C/A)]+[cr_{new}(\neg A)\times cr_{old}(C/\neg A)]\}$.[7]

Problem set 1:

- Tips for exercise 1:
    - Proof finding strategy: working backwards with definitions
        i. Take the equation you want to prove
        ii. Use definition to consider what you need to prove immediately beforehand
            1. For example, if you need to prove $cr(p|q) = cr(r|q)$, then work backwards

2. Ask yourself "What would I need to prove to get to $cr(p|q) = cr(r|q)$?" and apply the definitions to work backwards
3. Break it down using that definition
    a. $cr(p|q) = cr(r|q)$ then becomes:
        i. $\frac{cr(p\&q)}{cr(q)} = \frac{cr(r\&q)}{cr(q)}$
    b. You can then simply the equations you need to prove
        i. $cr(p\&q) = cr(r\&q)$

iii. Then, once you've simplified the equations, you can construct two partitions
1. For example, if p and q are incompatible, then you know a partition is $[p\&q, r\&q, \neg(p \lor r)\&q, \neg q]$

iv. Then, use the CrPP to show the relevant terms have the same probability
1. Steps 1-3 of proof 1a show you how to do this

- Other things you should know:
    - Adding fractions with common denominators:
        - $\frac{cr(A)}{cr(C)} + \frac{cr(B)}{cr(C)} = \frac{cr(A)+cr(B)}{cr(C)}$
    - Breaking down conditional credences involving disjunctions:
        - $cr(p \lor q|r) = \frac{cr((p \lor q)\&r)}{cr(r)} \neq \frac{cr(p \lor q\&r)}{cr(r)}$
    - Partitions with disjunctions of inconsistent propositions:
        - Suppose you have three propositions p, q, r and that p and q are inconsistent (incompatible?)
        - Then, these are two possible partitions:
            - $[p\&r, q\&r, \neg(p \lor q)\&r, \neg r]$
            - $[(p \lor q)\&r, \neg(p \lor q)\&r, \neg r]$
    - Proving inequalities:
        - Suppose you need to prove $cr(p) \leq cr(q)$ for some propositions p, q
        - You can get to this stage if you show that:
            1. $cr(p\&q) = cr(p)$ (this would be true if *p* implied *q*)
            2. $cr(q) = cr(p\&q) + cr(\neg p\&q)$ AND $cr(\neg p\&q)$ is greater or equal to zero, because then $cr(p\&q) \leq cr(q)$
        - You can use the CrPP to get to both of these stages

- Exercise 2:
    - It's party time, with Jeffrey conditionalization
    - Suppose you have the following distribution

party	*quiet*	
T	T	0.05
T	F	0.15
F	T	0.75
F	F	0.05

- Propositions:
    - *party* = *p* = there's a party next door
    - *quiet* = *q* = it's quiet next door
- Determine what your final credence in *party* is for each of the following:

a) First, you hear a rumor that there's a party next door, and $cr(party)$ increases to 0.6. But then it sounds quiet next door, and so $cr(quiet)$ increases to 0.8.

a. Answer

- New credences *after* first step:

party	*quiet*	Old credence	Weight	New credence (?)	New credence	
T	T	0.05	3	$cr_{new}(p) \times cr_{old}(p\&q\|p)$	0.15	0.6
T	F	0.15	3	$cr_{new}(p) \times cr_{old}(p\&\neg q\|p)$	0.45	
F	T	0.75	.5	$cr_{new}(\neg p) \times cr_{old}(\neg p\&q\|p)$	0.375	0.4
F	F	0.05	.5	$cr_{new}(\neg p) \times cr_{old}(\neg p\&\neg q\|p)$	0.025	

- New $cr(party)$ *after* second step:
  - **Important!! the new credences *after* the first step are now the *old* credences prior to the second step**

i. Jeffrey conditionalization:

$$cr_{new}(C) = cr_{new}(A) \times cr_{old}(C|A) + cr_{new}(\neg A) \times cr_{old}(C|\neg A)$$

ii. So:

$$cr_{new}(party) = cr_{new}(quiet) \times cr_{old}(party|quiet) + cr_{new}(\neg quiet) \times cr_{old}(party|\neg quiet) \approx 0.8 \times 0.286 + 0.2 \times 0.95 \approx 0.42$$

- So $cr(party) = 0.42$

b) First, it sounds quiet next door, and so $cr(quiet)$ increases to 0.8. But then you hear a rumor that there's a party next door, and $cr(party)$ increases to 0.6.

a. Answer: $cr(party) = 0.6$

- This illustrates the non-commutativity of Jeffrey Conditionalization