Section 6 Handout

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Things you need to know:

- Domain of objects:
  - The set of things the first-order language is about
  - Examples:
    - States: California, Texas…
    - Individuals: Donald Trump, Arnold Schwarznegar…
- Individual constants:
  - Symbols which stand for *specific* objects in the domain
  - Example notation:
    - *a, b, c, d…*
  - Examples:
    - *c* stands for California, *t* for Texas, *d* for Donald Trump…
- Individual variables:
  - Example notation:
    - *x, y, z…*
  - Meaning:
    - These are like place-holders for objects
    - Unlike constants, they do not apply to any particular object
    - (Their meaning and use will be clearer in the part on quantifiers)
- Relations:
  - These denote properties that may be possessed by some object or set of objects
  - They can vary in the number of objects they concern (see examples below)
  - Example notation:
    - *R, P, Q…*
  - Examples:
    - $D(x)$ = *x* is a democrat
    - $B(x, y)$ = *x* borders *y* (like how California borders Nevada)
    - $P(n, g, c)$ = *n* (standing for "Gavin Newsom") is the *g* (standing for "Governor") of *c* (standing for "California")
- Quantifiers:
  - Notation and meaning:
    - $\forall x\ldots$ = for all $x\ldots$
    - $\exists x\ldots$ = there exists an *x* such that…
  - Quantificational statements and translations
    - $\forall x(R(x) \rightarrow Q(x))$ = for all *x*, if *x* is an *R*, then *x* is a *Q* = All *Rs* are *Qs*
    - $\exists x\big(R(x) \land Q(x)\big)$ = there is some *x* such that *x* is an *R* and *x* is a *Q* = some *Rs* are *Qs*
  - Remember!
    - **Conjunctions** are generally used in claims with *some* quantifiers and **implications** are generally used in claims with *all* quantifiers
    - If a variable appears in a formula, there needs to be one quantifier for it

Practice exercises:

1. Formalize the following statements, and then explain why the third does or does not follow from the first two:
    a. Argument 1:
        i. All mumble-rappers are idiots
        ii. All idiots do not like EDM
        iii. Everyone who does not like EDM is a mumble rapper
    b. Argument 2:
        i. Not all mumble rappers are idiots
        ii. But some idiots are mumble rappers
        iii. Some mumble rappers are not idiots
2. For each of the following properties, provide a formula that says the relation R has the property:
    a. A relation is convergent just in case whenever $R(x, y)$ and $R(x, z)$ is true, there is some u such that $R(y, u)$ and $R(z, u)$ is true.
    b. A relation is serial just in case for every x, there is some y such that $R(x, y)$
    c. A relation is functional just in case for every x, if $R(x, y)$, then there is no distinct z such that $R(x, z)$.

Practice exercises:

1. Formalize the following statements, and then explain why the third does or does not follow from the first two:
    a. Argument 1:
        i. Formalization:
            1. All mumble-rappers are idiots
                a. $\forall x(M(x) \to I(x))$
            2. All idiots do not like EDM
                a. $\forall x(I(x) \to \neg E(x))$
            3. Everyone who does not like EDM is a mumble rapper
                a. $\forall x(\neg E(x) \to M(x))$
        ii. Explanation:
            1. The third does not follow from the first two. Every mumble rapper could be an idiot, and every idiot could not like EDM, but there could be other people who don't like EDM who aren't mumble rappers or idiots.[1]
    b. Argument 2:
        i. Formalization:
            1. Not all mumble rappers are idiots
                a. $\neg\forall x\ (M(x) \to I(x))$
            2. But some idiots are mumble rappers
                a. $\exists x\big(I(x) \land M(x)\big)$
            3. Some mumble rappers are not idiots
                a. $\exists x\big(M(x) \land \neg I(x)\big)$
        ii. Explanation:

[1] Note: this isn't quite true. As an objective matter of fact, everyone who doesn't like EDM is indeed an idiot.

1. The third claim follows from the first two, or—more specifically—just the first. If not all mumble rappers are idiots, then there must be some mumble rapper who is not an idiot.

2. For each of the following properties, provide a formula that says R has the property:
   a. A relation is convergent just in case whenever $R(x,y)$ and $R(x,z)$ is true, there is some u such that $R(y,u)$ and $R(z,u)$ is true.
      i. $\forall x \forall y\ \forall z((R(x,y) \wedge R(x,z)) \rightarrow \exists u(R(y,u) \wedge R(z,u)))$
   b. A relation is serial just in case for every x, there is some y such that $R(x,y)$
      i. $\forall x \exists y R(x,y)$
   c. A relation is functional just in case for every x, if $R(x,y)$, then there is no distinct z such that $R(x,z)$.
      i. $\forall x \forall y \forall z((R(x,y) \wedge R(x,z)) \rightarrow y = z)$ or…
      ii. $\forall x \forall y((R(x,y)) \rightarrow \neg \exists z(R(x,z) \wedge y \neq z))$