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Phil 184/284: Formal and Informal Epistemology
Section 8 Handout

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1. Ubiquity of probability
    a. Living a good life depends on you making good decisions
    b. Do you agree with the following?
        i. Your most important decisions are based on your credences
        ii. You are always less than *fully* certain about the most important topics in your life
            1. Clarification:
                a. You are certain of something when it is *impossible* for it to be false, given what you know
                b. Examples of things you're certain about:
                    i. $2 + 2 = 4$
                    ii. You are currently having a *perceptual experience* involving black and white
        iii. Our credences can be more or less trustworthy
2. Trustworthiness:
    a. Often can hope only for trustworthy credences/probabilities—not *certainty*
    b. Problem:
        i. Suppose you are 95% confident you will not get into Stanford
        ii. You get into Stanford
        iii. Were you dumb? Was your credence untrustworthy?
        iv. Not necessarily: improbable things happen sometimes
    c. When is a credence trustworthy?
        i. Calibration:
            1. One is calibrated just in case anything they attach a probability of *n* percent to is true approximately *n* percent of the time
            2. Example:
                a. You are calibrated only if anything you attach a probability of 80% is true approximately 80% of the time
    d. Questions:
        i. How calibrated are you?
        ii. How calibrated is your doctor, your advisor, the people who give you advice in your life?
        iii. Do you have decent evidence about *any* of these things?
3. Your credences are uncalibrated?
    a. Availability heuristic
        i. The psychological process whereby
            1. The probability of an event = the mental "availability" of that event—that is, the ease with which relevant instances come to mind
        ii. Explains false consensus effect:
            1. False consensus effect = overestimating the extent to which others are like you
    b. Motivational reasoning
    c. Jump to conclusions (JTC) bias
        i. We jump to conclusions without sufficiently considering:

        1. Alternative conclusions
        2. Argument for and *against* every possible conclusion
        3. The possibility we should collect more evidence

4. Problems:
    a. Problem case #1: Medical diagnosis
        i. Consider the following:
            1. Prior probability of the patient having the disease $P(D) = 0.01$
            2. Probability of the positive result given that the patient has the disease $P(T|D) = 0.792$
            3. Probability of the positive result given that the patient does not have the disease $P(T|\neg D) = 0.096$
        ii. Gut reaction:
            1. If you got positive on the test, what is the probability you have the disease?
            2. In other words, $P(D|T) =?$
        iii. Doctors' reactions:
            1. 95 out of 100 physicians estimated that probability was about 75%
    b. Problem case #2: Monty Hall problem
        i. Consider the following:
            1. A prize is randomly placed behind one of three doors: door A, door B and door C
            2. You are asked to select a door, although that door remains closed for the time being.
            3. Monty Hall knows where the prize is, and he will then open one of the other doors you did not chose
                a. If the door you first selected conceals the prize, he will open one of the other two doors at random.
                b. If the door you first selected does not conceal the prize, and one of the other two doors does, then he will open the unselected door that does not conceal the prize.
            4. Suppose you select door A, and then Monty Hall opens door C (which does not conceal the prize)
        ii. Gut reaction:
            1. Should you switch doors from your initial door (door A) and instead opt to open the other unopened door (door B)?
        iii. Most people's reaction:
            1. You should not switch doors

5. The correct answers:
    a. Bayes's theorem!
        i. Problem case #1:
            1. The correct answer:

$$P(D|T) =$$

$$\frac{P(T|D)P(D)}{P(T|D)P(D) + P(T|\neg D)P(\neg D)} =$$

$$\frac{(0.792)(0.01)}{(0.792)(0.01) + (0.096)(0.99)} =$$

$$0.077 \neq .75$$

ii. Problem case #2:
1. The correct answer:

$$P(B|c) =$$

$$\frac{P(c|B)P(B)}{P(c|A)P(A) + P(c|B)P(B) + P(c|C)P(C)} =$$

$$\frac{1 \times \frac{1}{3}}{0.5 \times \frac{1}{3} + 1 \times \frac{1}{3} + 0 \times \frac{1}{3}} =$$

$$\frac{2}{3} \neq \frac{1}{2}$$

b. Where do people go wrong?
i. Problem case #1:
1. People neglect:
a. Prior probabilities $P(h)$
b. Likelihood of the evidence given the alternative $P(e|\neg h)$
ii. Problem case #2:
1. People neglect:
a. Likelihood of the evidence given the hypothesis and the alternative $P(e|h) \& P(e|\neg h)$

6. The solution:
a. J-Cond:

$$cr_{new}(h) = cr_{new}(e) \times cr_{old}(h|e) + cr_{new}(\neg e) \times cr_{old}(h|\neg e)$$

where:

$$Bayes' theorem\colon\ cr_{old}(h|e) = \frac{cr_{old}(e|h)cr_{old}(h)}{cr_{old}(e|h)cr_{old}(h) + cr_{old}(e|\neg h)cr_{old}(\neg h)}$$

b. Rules:
i. *Think* in terms of probability and calibration
ii. Hypothesis and evidence identification:
1. Always consider:
a. As many plausible alternatives as possible
b. Evidence for and *against* the alternatives
c. The possibility we should collect *more* evidence before assigning probabilities
iii. Hypothesis evaluation:
1. Always consider:
a. Prior probabilities
b. Probability of the evidence given the hypothesis
c. Probability of the evidence given the *alternative* hypotheses
iv. Use *frequencies* to inform all of these things (and not availability, your desires...)
1. Frequency = how often something happens, or the proportion of the time it happens
a. E.g. the proportion of the time door B will be opened if door C conceals the prize