Draft
Ad Hocness, Accommodation and Consilience: A Bayesian Account

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Abstract

We all make judgments about what is true or false, probable or improbable. Scientists do so too. And in the process, we frequently appeal to concepts such as evidential support or explanation. Bayesian philosophers of science have given illuminating formal accounts of these concepts. This paper follows in their footsteps, providing a novel formal account of three concepts: successful accommodation of evidence, ad hocness, and consilience or—as it is sometimes called—unification. Using these accounts, I also provide a new Bayesian analysis of how someone such as Darwin hypothetically could have reasoned in favor of evolution over special creationism. Lastly, I explore how these accounts relate to other topics and accounts in philosophy, and I chart out some areas for further research.

1. Introduction

Everyday, we make important epistemic judgments about what is true or false, or about what is probable or improbable. This is true in mundane ordinary contexts: we make important judgments about whether a particular medication is probably safe to take, for instance. But it is also true in scientific contexts: for example, science accepts the truth of evolutionary theory, and this has had revolutionary implications for how we understand ourselves, our historical origins and our place in the world.

In making these judgments, we often appeal to various concepts, such as evidential support, explanation or causation. For example, we might believe evolutionary theory is true because it *explains* various facts, such as the genetic similarity between species, and each of these facts thereby provides *evidential support* for the theory.

Bayesian philosophers of science have attempted to explicate concepts like these. Consider, for example, the concept of evidential support. According to one prominent account of this concept, some evidence e supports or "confirms" a hypothesis h if and only if e raises the probability of h, and it raises the probability as such when $P(h|e) > P(h)$ (where $P(h|e)$ is the probability of h conditional on e and $P(h)$ is the probability of h prior to the receipt of the evidence e).[1] This account represents an attempt to explicate one concept—evidential support—in terms of a quantitative account—probability-raising.

Note that these concepts are things which we might initially have only *qualitative judgments about*. Presumably scientists have judged for centuries that evidence can *support* one hypothesis or another, but they presumably did not always think of this "support" in precise quantitative terms—well, at least not before they became conceptually equipped to think in terms of probability.

For the most part, the project of Bayesian philosophy of science has been to explicate imprecise, qualitative judgments about such concepts in more precise, quantitative terms.

[1] Jan Sprenger and Stephan Hartmann, *Bayesian Philosophy of Science* (Oxford; New York: Oxford University Press, 2019).

The task of this paper is to then extend this project to provide a unified account of three further concepts: successful accommodation, ad hocness and consilience. Here, “successful accommodation” refers specifically to when a hypothesis successfully accommodates some evidence. This typically looks like an attempt to reconcile the truth of the hypothesis with what would have otherwise been counter-evidence to that hypothesis. The second concept—ad hocness—then concerns unsuccessful attempts to save a hypothesis from such counter-evidence. The third concept—consilience—concerns when a theory successfully explains multiple different kinds of evidence.[2] This consilience is also sometimes called *unification*.[3] The account of these concepts is unified in the following sense: a hypothesis successfully accommodates some evidence just in case it does not appeal to ad hoc auxiliary hypotheses, and a hypothesis often successfully unifies or consiliates a body of evidence when alternative hypotheses would make ad hoc appeals to auxiliary hypotheses to try accommodate that body of evidence.

The structure of this paper is as follows. In part II, I give a Bayesian account of successful accommodation and ad hocness. In part III, I give a Bayesian account of consilience, and show how it relates to the account of ad hocness. In part IV, I explore how these accounts relate to the existing philosophical literature, including existing accounts of unification. There, I will argue that, unlike previous literature, the account in this paper provides some simple and interrelated quantitative formalisms for assessing ad hocness and unification. And in part V, I explore some unresolved questions for future research.

2. Successful Accommodation and Ad Hocness

In science and in everyday life, we typically consider a range of hypotheses. In science, some historically prominent examples of hypotheses are the hypotheses that Newtonian mechanics is provides a true account of physics, that the species evolved through natural selection and that the earth is older than 12,000 years old. In everyday life, we might also entertain more mundane hypotheses about various things: take, for instance, the hypothesis that some medication is safe for you to consume, or that it rained outside while you were in a movie theater.

Sometimes, these hypotheses confront counter-evidence. In science, for example, Mercury’s orbit is now regarded as counter-evidence to Newtonian mechanics. This was because Newtonian mechanics predicted that Mercury should orbit the sun in a particular way, but this was different to Mercury’s actual orbit. Then there’s more mundane everyday examples of counter-evidence: you might experience adverse symptoms after taking some medication, and you might think this is counter-evidence to your hypothesis that the medication is safe.

Hypotheses frequently encounter counter-evidence, and we often try to reconcile the hypotheses with the counter-evidence.

Typically, we do this by appealing to *some* other hypothesis aside from the one we are mainly interested in. For example, to reconcile the safety of the medication with the adverse symptoms, we might appeal to *another* hypothesis: the hypothesis that something else caused the symptoms, such as an allergy to your

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[2] Robert Nola, “Darwin’s Arguments in Favour of Natural Selection and Against Special Creationism,” *Science & Education* 22, no. 2 (February 16, 2013): 149–71, https://doi.org/10.1007/s11191-010-9314-3; Larry Laudan, “William Whewell on the Consilience of Inductions,” ed. Sherwood J. B. Sugden, *Monist* 55, no. 3 (August 1, 1971): 368–91, https://doi.org/10.5840/monist197155318; Richard R. Yeo, *Defining Science : William Whewell, Natural Knowledge, and Public Debate in Early Victorian Britain* (Cambridge: Cambridge University Press, 1993), https://searchworks.stanford.edu/view/2759046.

[3] Wayne C Myrvold, “A Bayesian Account of the Virtue of Unification*,” *Philosophy of Science*, vol. 70, 2003; Philip Kitcher, “Explanatory Unification,” *Philosophy of Science* 48, no. 4 (December 1981): 507–31, https://doi.org/10.1086/289019; Thomas Blanchard, “Bayesianism and Explanatory Unification: A Compatibilist Account,” *Philosophy of Science* 85 (2018): 682–703.

dinner. Hypotheses like these are often appealed to in order to "explain away" the counter-evidence—as some scholars say.

Philosophers of science call such hypotheses *auxiliary hypotheses*—they are hypotheses that are distinct from the main hypothesis that one is interested in, but they nevertheless have implications for how the evidence relates to that main hypothesis.

An auxiliary hypothesis is typically called "ad hoc" when it represents an unsatisfactory attempt to save a theory from some counter-evidence—although there are different accounts is to precisely how these attempts are unsatisfactory.

An example of this is Newtonian mechanics and the orbit of Mercury. Let us consider it in more detail. In 1859, Urbain Le Verrier recognized that Mercury orbited the sun in a way which conflicted with the predictions of Newton's law of physics. In an attempt to reconcile Mercury's orbit with Newton's laws, scientists advanced various hypotheses. Le Verrier himself also advanced one: that there was an undetected planet that had a gravitational influence on the orbit of Mercury, and this influence supposedly explained away the apparent conflict between Newton's laws and Mercury's orbit. However, such as planet was never found. A range of other hypotheses were proposed, each with their problems. Various commentators appraised these hypotheses as "ad hoc": they attempted to salvage theories in ways that were in some sense unsatisfying.[4]

However, ad hoc hypotheses are not confined only to discussions in science. The comedian Bill Hicks relates a conversation with a young earth creationist who believed the world was 12,000 years old.[5] Hicks asked him what he thought about dinosaurs and dinosaur fossils. The creationist reportedly replied, "God put those there to test our Faith." In this story, we have an example of an ad hoc hypothesis in an everyday context: someone tries to salvage some hypothesis (young earth creationism) by appealing to another one (God testing our faith). And we might think there is something bothering about this ad hoc hypothesis, as Hicks himself does:

> Does that bother anyone here? The idea that God might be f**king with our heads… that he's running around, burying skulls [saying] 'We'll see who believes in me now?'

Of course, who knows whether this conversation occurred in the way that Hicks describes it, but the point is that we are all familiar with attempts to save beliefs with ad hoc hypotheses like these.

However, attempts to save a hypothesis from counter-evidence are not always ad hoc. The story of Le Verrier himself offers an example of this. Well before discovering the conflict between Newtonian mechanics and Mercury's orbit, he had discovered another tension: Uranus also did not orbit the sun in the way that scientists expected if Newtonian mechanics was true. As a result, some suspected that Newtonian mechanics was false. However, Le Verrier thought that a solution to this conflict lied elsewhere: he hypothesized that there was a specific undetected planet that affected Uranus' orbit, one which could resolve the tension in a Newtonian framework. He then deduced that this undetected planet could be observed at a particular time and location in the solar system. When astronomers tested this prediction, they did indeed find the planet that Le Verrier had postulated. This planet is now known as Neptune.

The postulation of Neptune is now regarded as one of science's best success stories—far from an ad hoc attempt to save a theory from some counter-evidence.[6]

Clearly, then, some appeals to auxiliary hypotheses successfully accommodate the evidence while others do not. And when these auxiliary hypotheses do not succeed, we often call them "ad hoc".

What then distinguishes successful attempts to save a theory from those that are ad hoc? More precisely, under what conditions is an auxiliary hypothesis ad hoc?

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[4] Stephen G Brush, "Prediction and Theory Evaluation: The Case of Light Bending," *Science* 246, no. 4934 (1989): 1124–29.
[5] Bill Hicks, "Dinosaurs," *Arizona Bay* (album, Austin, TX: Rykodisc), 1997.
[6] Colin Howson and Peter Urbach, *Scientific Reasoning: The Bayesian Approach*, 3rd ed (Chicago: Open Court, 2006).

There are various candidate answers to this question, some of which I explore in Part IV. However, I want to focus on one candidate: a Bayesian account.

2.1. The Likelihood-Prior Trade-Off

In this section, I will outline a Bayesian account of when attempts to accommodate auxiliary hypotheses are successful versus when they are not—and, in particular, when they are 'ad hoc'.

An attempt to accommodate counter evidence typically appeals to an auxiliary hypothesis to save some main hypothesis. By this, I mean that when someone confronts counter-evidence to some main hypothesis, they may start to entertain—in their mind and perhaps also in their speech—some other auxiliary hypothesis. When Newtonian mechanics was threatened by Mercury's orbit, Le Verrier appealed to the existence of Vulcan. When Newtonian mechanics was threatened by Uranus' orbit, Le Verrier appealed to the existence of Neptune. And when the creationist's worldview was threatened by the existence of dinosaur fossils, he appealed to God testing our faith. In each case, some other hypothesis—distinct from the main hypothesis—is entertained in one's mind or discussions to accommodate the evidence.

We can use probabilistic notation to formalize this. We can symbolize the main hypothesis with the letter *h*, the counter-evidence with *e* and the auxiliary hypothesis with *a*. For example, *h* is the hypothesis that the earth is 12,000 years old, *e* is the fact that dinosaur fossils exist, and *a* is the hypothesis that God put fossils here to test our faith. An appeal to an auxiliary hypothesis means that we are no longer entertaining how h fares *by itself* with respect to *e*, but rather how the *conjunction of* h *and* a fares with respect to *e*.

Typically, the aim of the appeal is to show that the counter-evidence is more probable given the conjunction of h and a than it appears to be when we consider just h by itself. For example, even though Uranus' orbit did not at first seem probable if Newtonian mechanics was true, it certainly seemed more probable if Newtonian mechanics was true *and* there was another planet which influenced Uranus' orbit in accordance with Newtonian mechanics. Here, *h* is the main hypothesis that Newtonian mechanics is true, *a* is the auxiliary hypothesis that some other planet exists with specific properties, and *e* is the orbit of Uranus. Let us use the notation $P(e|h)$ to denote the probability that Uranus would orbit the way it did given that Newtonian mechanics is true. And let $P(e|h\&a)$ denote the probability of Uranus' orbit given that Newtonian mechanics is true *and* there is another planet which influenced Uranus' orbit. In probabilistic terms, the appeal to the auxiliary hypothesis aimed to show this: even if $P(e|h)$ appeared low and Uranus' orbit did not at first seem probable if Newtonian mechanics was true, $P(e|h\&a)$ was much higher and the orbit was more probable given that Newtonian mechanics is true *and* there was another planet which influenced Uranus' orbit in a specific way.

Put simply, attempts to accommodate the counter-evidence typically amount to the claim that $P(e|h\,\&\,a)$ is much higher than $P(e|h)$ such that $P(e|h) \ll P(e|h\,\&\,a)$ for some main hypothesis *h*, counter-evidence *e* and auxiliary hypothesis *a*. Technically, the term for conditional probabilities like these are 'likelihoods'—that is, the probability of some evidence given some hypothesis (or hypotheses). Appeals to auxiliary hypotheses then have the potential advantage of raising the *likelihood* of the evidence in this particular sense.

However, such appeals are not without their costs. The reason for this is that the probability of two hypotheses being simultaneously true is almost always lower than the probability of just one of them being true. For example, no matter how probable Newtonian mechanics was at the time, one could not be as confident that Newtonian mechanics was true *and* another planet like Neptune existed. This is because the truth of this conjunction relies not only on the probability that Newtonian mechanics is true, but it further relies on the also less-than-certain probability that the other planet exists. And uncertainty coupled with even more uncertainty in this way can only increase one's uncertainty: the probability of the conjunction has to be lower than the probability of either one of its conjuncts. There is, however, an exception to this when one is certain that both hypotheses are true, or they are certain that one hypothesis is true conditional on the other, or they are certain that at least one hypothesis is false. Otherwise, there is always a cost in the sense that the probability of the conjunction is lower than the probability of the conjunct.

So we have seen that an attempt to accommodate counter-evidence will typically raise the likelihood of the evidence by appealing to some auxiliary hypothesis *a* such that $P(e|h) \ll P(e|h\&a)$.

And this typically comes with a cost because the probability of h & a is almost always lower than the probability of h by itself. In other words, often $P(h) > P(h\&a)$ where $P(h)$ is the probability of the hypothesis and $P(h\&a)$ is the probability of the conjunction of hypothesis and the auxiliary hypothesis. This also is true when we interpret $P(h)$ and $P(h\&a)$ as *prior* probabilities—that is, as the probabilities of the hypotheses prior to receiving some evidence. For example, Le Verrier might have had a probability of 0.8 that Newtonian mechanics was true *prior* to learning about Uranus' orbit, but this probability might have increased to 0.9 *after* he updated on the evidence that Neptune exists and could be deduced from Newtonian mechanics and Uranus' orbit. Here, the prior probability is 0.8 while the so-called *posterior* probability is 0.9. The claim, then, is that $P(h) > P(h\&a)$ often holds for prior probabilities too.

Consequently, in a sense, appeals to auxiliary hypotheses often raise the likelihood of the evidence such that $P(e|h) < P(e|h\&a)$, but they lower the prior probability of the relevant propositions in the sense that $P(h) > P(h\&a)$. We can articulate this more formally with a principle:

Likelihood-Prior Trade-Off principle:

(1) If $P(e|h) < P(e|h\&a)$, then $P(h) > P(h\&a)$ where $0 < P(h)$and $P(a|h) < 1$.

This is a simple principle, the truth of which will be obvious to many. But for the sake of completeness, proof of the principle is found in the appendix regardless. In any case, the point is that, according to the principle, attempts to accommodate counter-evidence come with a trade-off.

And sometimes this tradeoff is unsuccessful.

Consider Hicks' creationist again. The creationist endorsed the main hypothesis *h*—that the world is 12,000 years old. He was then confronted with the counter-evidence *e* of dinosaur fossils. He then appealed to an auxiliary hypothesis *a* to raise the likelihood of this evidence: God put dinosaur fossils on earth to test our faith. And this did successfully raise the likelihood of the evidence: after all, *if* we suppose God put dinosaur fossils there to test our faith, *then* we would not be so surprised to see dinosaur fossils, even if the world is 12,000 years old. The problem with the creationist's reasoning, however, is that it raised the likelihood of the evidence only by appealing to an auxiliary hypothesis which—to many of us at least—is obviously implausible. Consequently, the likelihood was raised, but only by lowering the relevant prior probability: we are now considering not just the already improbable proposition that the world is 12,000 years old, but we are now considering the much less probable conjunction of that proposition with the claim that God put dinosaur fossils on earth to test our faith. (Of course, for the creationist, the prior probability of God testing our faith may not be so low, and I shall discuss issues with rational constraints on prior probabilities in part IV of this paper.)

So some tradeoffs are not successful, but others are successful. For instance, suppose you develop some adverse health symptoms after taking some medication. You initially believed the hypothesis that the medication was safe, but if this was the case, then it seems to you that the symptoms would be unlikely. However, you might remember that you had an allergen in your dinner, and so you successfully accommodate this evidence by appealing to the auxiliary hypothesis that you are having an allergic reaction to your dinner instead. By your lights, this is a successful accommodation: it makes the symptoms more likely, and it does so by appealing to an auxiliary hypothesis that—we suppose—you have good evidence to be confident in. In this sense, the tradeoff is successful: it raises the likelihood of the evidence, but without drastically reducing the prior probability of the propositions you are entertaining.

More formally, then, when are appeals to auxiliary hypotheses successful? In other words, when is the likelihood-prior trade-off worth it in formal terms?

2.2. *Successful Accommodation and Ad Hocness*

I will offer a couple of accounts in this paper, each focused on specific contexts.

Such accounts, however, explicitly mention the *comparative* nature of inquiry: when we are considering the probability of some main hypothesis h_1, we are always considering it in *comparison* to some *alternative* hypothesis h_2. And we can see that this is true because often this alternative hypothesis is simply the hypothesis that the main hypothesis is false: if the main hypothesis is h_1, then the alternative hypothesis h_2 is such that $\neg h_1 \equiv h_2$. But sometimes the alternative is just one of a variety of ways in which the main hypothesis could be false: h_1 might be that Newtonian mechanics is true, and h_2 might be an alternative hypothesis that relativity theory is true, even if there are further alternatives such as the hypothesis h_3 that some as yet undiscovered quantum theory is true. In this case, we could suppose $\neg h_1 \equiv (h_2 \vee h_3)$.

Given the comparative nature of inquiry, we can articulate an account of successful accommodation as such. Let h_1 and h_2 be two mutually exclusive hypotheses and let *e* be some evidence such that *e* disconfirms h_1 and confirms h_2 (i.e. $\frac{P(h_1)}{P(h_2)} < \frac{P(h_1|e)}{P(h_2|e)}$). Then,

Account of Successful Accommodation:

(2) An auxiliary hypothesis *a* successfully accommodates the evidence with h_1 just in case:

$$\frac{P(h_1)}{P(h_2)} \leq \frac{P(h_1 \& a|e)}{P(h_2|e)}$$

Put informally, this principle compares the relative probabilities of h_1 and h_2. On the left side, it compares their relative probabilities *without* conditioning on the evidence. On the right side, it compares their relative probabilities *given the evidence*, *but* with h_1 conjoined to *a*. The account then says that h_1 and *a* successfully accommodate the evidence just in case the conjunction of h_1 and *a* fares at least as well in the light of the evidence (relative to h_2) as h_1 fared relative to h_2 prior to conditioning on the evidence. Of course, one could think of other possible accounts of successful accommodation, and I will discuss one of these later, but this account is adequate for our current purposes.[7]

If we accept this account, then, we can derive the following theorem from the probability calculus:

Theorem of Successful Accommodation:

(3) $\frac{P(h_1)}{P(h_2)} \leq \frac{P(h_1 \& a|e)}{P(h_2|e)}$ iff $P(e|h_2) \leq P(e|h_1 \& a)P(a|h_1)$

Proof of the theorem is in the appendix. In words, what this says is that h_1 and *a* successfully accommodate the evidence only when the likelihood of the evidence given h_2 is less than the product of two terms: 1) the likelihood of the evidence given h_1 and the auxiliary hypothesis, and 2) the probability of the auxiliary hypothesis given h_1. With this theorem, we have an easy way to see whether the cost of the likelihood-prior trade-off is worth it.

We can now apply this to our examples, starting with Hicks' creationist.

[7] In particular, one might think that a better account is $\frac{P(h_2)}{P(h_1)} \leq \frac{P(h_2 \& a|e)}{P(h_1 \& a|e)}$. I will discuss this later on.

2.3. *Illustrative Application: Hicks' Creationist*

Let the main hypothesis h_1 be that the world is 12,000 years old. Let the alternative hypothesis h_2 be that the world is 4.5 billion years old, as well as the background theory that serves as the foundation for the current scientific understanding of the earth's age, origin and development. (And we can suppose this background theory is what allows us to assign a value to $P(e|h_2)$.)

In this case, the creationist was then confronted with the counter-evidence *e* of dinosaur fossils.

This evidence is not so improbable if h_2 is true: after all, if the earth is very old, then we would not be surprised to see such dinosaur fossils. For illustration's sake, let us say then that $P(e|h_2) = 0.5$. The specific value here is not so important here because, as we will see, all that matters is that the value is higher relative to other specific probabilities, including $P(e|h_1)$.

And on that note, it clearly seemed to Hick that $P(e|h_1)$ is low: if the earth is only 12,000 years old, then it seems improbable that we would observe dinosaur fossils that make it look much, much older. As Hicks describes it, the creationist then appealed to an auxiliary hypothesis *a* to save his preferred theory: God put those dinosaur fossils on earth to test our faith. Arguably, the probability of such fossils is then much higher given the conjunction of h_1 and this auxiliary hypothesis: if the earth was 12,000 years old *and* God really did put dinosaur fossils here to test our faith, then of course we would see such dinosaur fossils. $P(e|h_1\&a)$ is then very high—arguably a probability of 1, in fact!

So $P(e|h_1) < P(e|h_1\&a)$, and we have a classic case where one attempts to save a hypothesis by appealing to an auxiliary hypothesis to raise the likelihood of the evidence.

But as we have seen, this comes at a cost, and—in this case—a dire one. Even though $P(e|h_1\&a)$ is high, the probability of the auxiliary hypothesis is very low: it seems very implausible that God would put dinosaur fossils here just to test our faith. In this vein, Hicks expresses his incredulity at this auxiliary hypothesis: "Does that bother anyone here? The idea that God might be f**king with our heads… that he's running around, burying skulls [saying] 'We'll see who believes in me now?'"

I will assume that the reader shares an incredulity towards this hypothesis, in which case we can suppose $P(a)$ is very low—for illustrative purposes, say, $P(a) = 0.01$. And let us suppose that this is still true even if we suppose that the world is 12,000 years old. In other words, nothing about the world being 12,000 years old would by itself us to expect that God would want to test our faith *prior* to learning of the existence of dinosaur fossils. Then, $P(a) = P(a|h_1) = 0.01$.

In a sense, perhaps even a creationist would agree with this prior probability too. If a creationist never knew of the existence of dinosaur fossils, they might agree that God is unlikely to do something as extreme as test our faith with misleading evidence about the age of the earth. If you asked them about this in advance of them learning the fact, they might be even quite confident that a loving God would not mislead his creatures so severely. But of course, for various reasons, one might not let their prior probabilities affect their later credences in appropriate ways: one might reason backwards from their desired conclusions to reject any premise or counter-evidence they want, all without regard to any prior probabilities. (Again, though, we will discuss the topic of rational constraints on prior probabilities in Part IV.)

But in any case, many of us would agree that $P(a)$ is very low, and we can proceed to analyze how we might make sense of the creationist's ad hoc maneuver in a Bayesian framework.

Within our framework, we can then see whether the attempt to save the creationist hypothesis is successful. Recall that according to the account of successful accommodation, an auxiliary hypothesis *a* successfully accommodates the evidence with h_1 just in case:

$$\frac{P(h_1)}{P(h_2)} \leq \frac{P(h_1\&a|e)}{P(h_2|e)}$$

In other words, h_1 and *a* successfully accommodate the evidence just in case the conjunction of h_1 and *a* fares at least as well in the light of the evidence (relative to h_2) as h_1 fared relative to h_2 prior to conditioning on the evidence. According to the theorem of successful accommodation, then, this is true just in case $P(e|h_2) \leq P(e|h_1\&a)P(a|h_1)$.

We can then plug in our values above to see whether the attempt does indeed accommodate the evidence:

$$P(e|h_2) = 0.5 > P(e|h_1\&a)P(a|h_1) = (1)(0.01) = 0.01$$

Here, we can see that the attempt is not successful: it does indeed raise the likelihood of the evidence, but only by appealing to an implausible auxiliary hypothesis. Since the auxiliary hypothesis is not successful as such, we can then say that it is ad hoc. And intuitively, this makes sense: some evidence should undermine a hypothesis when that hypothesis can raise the comparatively low probability of the evidence only with the help of improbable auxiliary hypotheses. The Bayesian analysis of this appears intuitively plausible.

2.4. Limitations of the Account: Sequential Updating

So we have seen how the formalism applies to the creationist case. That case involves one update: there, you have the prior probabilities, you update on the evidence, and the formalism specifies that an appeal to the auxiliary hypothesis is ad hoc because the auxiliary hypothesis has a low probability. We can call this kind of updating *one-shot updating*.

The cases of Uranus and Mercury, in contrast, involve *sequential updating*. In these cases, you have the prior probabilities, you update on the evidence about the anomalous perturbation of a planet and *then you update again*, but this time on further information about the auxiliary hypotheses. In the case of Uranus, the scientists updated on the evidence that the planet orbited in an anomalous way, and then they updated again once they found that Neptune existed and could indeed explain Uranus' orbit. In the case of Mercury, the scientists updated on the evidence that the planet orbited in an anomalous way, and then they updated again once they found that Vulcan was unlikely to exist. In each of these cases, further investigation occurred after the evidence was received, and subsequent updating served to either vindicate the auxiliary hypothesis or to condemn it.

In general, the formalism does not fully illuminate cases where such sequential updating takes place. This is not necessarily a fault of the formalism; instead, it merely that the formalism applies to a limited domain which does not include sequential updating.

To see its limits, let us go back to the basics using the case of Uranus as an example.

In philosophy, Bayesians typically give an account of Bayesian probability that looks something like the following. First, the probability is defined over a space of outcomes that are constructible from a formal language. A formal language is a set of symbols, and, in this case, the symbols represent propositions of interest. In our example, suppose our formal language features the just following atomic propositions: h_N which stands for the main hypothesis that Newtonian mechanics is true, e_U which stands for the anomalous orbit of Uranus and a_N which stands for the auxiliary hypothesis that a planet like Neptune exists. From these atomic propositions, we can construct the space of possible outcomes and assign a prior probability measure for each of these outcomes. Let us use the notation s_i to represent the i-th outcome in the space of possible outcomes, and let $P_p(.)$ symbolize this prior probability measure.

We can then depict the probability assigned to these outcomes with the below table:

<table>
<tr><th>Possible outcome</th><th>Conjunction of atomic propositions that are true in that outcome</th><th>Prior probability measure $P_p(.)$</th><th colspan="2">$P_p(.)$ over collections of outcomes</th></tr>
<tr><td>s_1</td><td>h_N & e_U & a_N</td><td>$.018$</td><td rowspan="2">$.02$</td><td rowspan="4">$.9$</td></tr>
<tr><td>s_2</td><td>h_N & e_U & $\neg a_N$</td><td>$.002$</td></tr>
<tr><td>s_3</td><td>h_N & $\neg e_U$ & a_N</td><td>0</td><td rowspan="2">$.88$</td></tr>
<tr><td>s_4</td><td>h_N & $\neg e_U$ & $\neg a_N$</td><td>$.88$</td></tr>
<tr><td>s_5</td><td>$\neg h_N$ & e_U & a_N</td><td>$.0005$</td><td rowspan="2">$.05$</td><td rowspan="4">$.1$</td></tr>
<tr><td>s_6</td><td>$\neg h_N$ & e_U & $\neg a_N$</td><td>$.0495$</td></tr>
<tr><td>s_7</td><td>$\neg h_N$ & $\neg e_U$ & a_N</td><td>$.005$</td><td rowspan="2">$.05$</td></tr>
<tr><td>s_8</td><td>$\neg h_N$ & $\neg e_U$ & $\neg a_N$</td><td>$.045$</td></tr>
</table>

In the above probability table, we can see the probabilities assigned to the space of possible outcomes and, by implication, to the atomic propositions that correspond to collections of outcomes. For example, the atomic proposition for Newtonian mechanics h_N is true just in case one of outcomes in the set of outcomes $\{s_1, s_2, s_3, s_4\}$ is true. Consequently, $P_p(h_N) = P_p(h_N \,\&\, e_U \,\&\, a_N) + P_p(h_N \,\&\, e_U \,\&\, \neg a_N) + P_p(h_N \,\&\, \neg e_U \,\&\, a_N) + P_p(h_N \,\&\, \neg e_U \,\&\, \neg a_N) = .018 + .002 + 0 + .88 = 0.9$

On the Bayesian account, such a prior probability distribution models some agent's degrees of belief or confidence at the *beginning* of the story of Uranus. According to the prior probability distribution, the agent's confidence in Newtonian mechanics is high since $P_p(h_N) = 0.9$. In contrast, the prior probability of Uranus orbiting in an anomalous way is low since $P_p(e_U) = 0.07$ and the prior probability of a planet like Neptune is very low since $P_p(a_N) \approx .02$. In particular, if Newtonian mechanics is true, then the anomalous orbit of Uranus is highly unlikely, since $P_p(e_U|h_N) \approx 0.02$. Such is the state of the agent at the beginning of the story of Uranus.

However, in the story, then scientists discover that Uranus is orbiting in an anomalous way which is not expected given Newtonian mechanics. In other words, they discover that e_U is true.

According to the Bayesian story, the agent must then update their probability distribution to another distinct "*posterior*" probability distribution. We can symbolize this distribution with the notation $P_e(.)$, with the *e* subscript to reflect the fact that it is after or posterior to the receipt of the *evidence*. Bayesians then endorse a principle of conditionalization, according to which $P_e(q) = P_p(q|e) = \frac{P_e(q \,\&\, e)}{P_e(e)}$ for any proposition *q* given some evidence *e*. In other words, the agent's posterior degree of belief in any proposition should be equal to their earlier degree of belief in that proposition conditional on the evidence.

We then obtain a different distribution of probabilities as follows:

Possible outcome	Conjunction of atomic propositions that are true in that outcome	Prior probability measure $P_p(.)$	Posterior probability measure $P_e(.)$
s_1	h_N & e_U & a_N	$.018$	$.26$
s_2	h_N & e_U & $\neg a_N$	$.002$	$.03$
s_3	h_N & $\neg e_U$ & a_N	0	0
s_4	h_N & $\neg e_U$ & $\neg a_N$	$.88$	0
s_5	$\neg h_N$ & e_U & a_N	$.0005$	$.01$
s_6	$\neg h_N$ & e_U & $\neg a_N$	$.0495$	$.7$
s_7	$\neg h_N$ & $\neg e_U$ & a_N	$.005$	0
s_8	$\neg h_N$ & $\neg e_U$ & $\neg a_N$	$.045$	0

The receipt of the evidence then affects the agent's confidence in Newtonian mechanics. They were initially quite confident in Newtonian mechanics, since $P_p(h_N) = .9$. In contrast, they become substantially less confident in Newtonian mechanics after receiving the anomalous evidence, since $P_e(h_N) = .29$. This is partly because of the fact that if Newtonian mechanics was true, the agent would not have expected the anomalous precession of Uranus since $P_p(e_U|h_N) \approx 0.02$.

The formalism can adequately explicate the sense in which the appeal to a planet like Neptune would have initially been ad hoc. This is because, at first, there was no evidence for another planet that influenced Uranus' pull, and the agent was initially confident there was no such planet. Consequently, the appeal to Neptune does not successfully accommodate the evidence at first, and this jibes with the formalism:

$$P_p(e_U|\neg h_N) = .5 > P_p(e_U|h_N \& a_N)P_p(a_N|h_N) = (1)(0.02) = 0.02$$

However, the formalism does not apply to the rest of the story which features sequential updating. Once the agent has updated on the evidence, a new probability function $P_e(.)$ then models their degrees of confidence. After further investigation, however, scientists discover that Neptune exists, and the agent must again update their degrees of belief to a new probability function. Let us symbolize this new probability function with $P_a(.)$—the subscript a indicating that the function models the agent's degrees of belief *after* they have learned that the *auxiliary* hypothesis a_N is true.

The probability novice may assume that the way to determine ad hocness is to plug values into a formula like this:

$$P_p(e_U|\neg h_N) > P_p(e_U|h_N \& a_N)P_a(a_N|h_N)$$

Here, they would be comparing the likelihood of the evidence according to the *prior* distribution $P_p(.)$ with the probability of the auxiliary hypothesis according to the later distribution $P_a(.)$.

The problem with this attempt is that the theorem of successful accommodation does not by itself relate *two distinct probability distributions*—one at an earlier time and one at a later time. And no similar theorem which relates different distributions follows straightforwardly from the probability calculus. For that reason, the theorem does not apply to comparisons involving sequential updating. It only concerns one probability distribution—not the relations among many such distributions.

Furthermore, any attempt to use the theorem with just one of the distributions does not capture the intuition that the appeal to Neptune is no longer ad hoc once it has been discovered. For example, the probability distribution $P_p(.)$ does not capture the sense in which the auxiliary hypothesis becomes probable later on, simply because a_N always has a low probability on that distribution. The probability distribution $P_a(.)$ does capture the sense in which the auxiliary hypothesis is probable later on, but by that point in time, the evidence has been updated on, and the likelihoods are equal on both the hypothesis and its negation—i.e. $P_a(e_U|h_N) = P_a(e_U|\neg h_N) = 1$. There is then no point in comparing the likelihoods with the theorem of successful accommodation, since the likelihoods are the same.

For these reasons, the theorem of successful accommodation does not illuminate such cases of sequential updating, even though it may illuminate parts of them (such as the one-shot update at the beginning when an appeal to another planet was arguably ad hoc in Uranus' case).

However, this is not to say the Bayesian offers no guidance here, for the standard tools of Bayesian updating can still apply. There, the probability of any proposition q after learning some proposition r is such that $P_a(q) = P_e(q|r) = \frac{P_e(q \,\&\, r)}{P_e(r)}$. Let us see how this applies. Once the scientists have learned that a is true, their credences take on a new probability distribution $P_a(.)$ in accordance with conditionalization, and we can update the table to reflect this.

Possible outcome	Conjunction of atomic propositions that are true in that outcome	Prior probability measure $P_p(.)$	Posterior probability measure $P_e(.)$	Posterior probability measure $P_a(.)$
s_1	$h_N \& e_U \& a_N$	$.018$	$.26$	0.96
s_2	$h_N \& e_U \& \neg a_N$	$.002$	$.03$	0
s_3	$h_N \& \neg e_U \& a_N$	0	0	0
s_4	$h_N \& \neg e_U \& \neg a_N$	$.88$	0	0
s_5	$\neg h_N \& e_U \& a_N$	$.0005$	$.01$	0.04
s_6	$\neg h_N \& e_U \& \neg a_N$	$.0495$	$.7$	0
s_7	$\neg h_N \& \neg e_U \& a_N$	$.005$	0	0
s_8	$\neg h_N \& \neg e_U \& \neg a_N$	$.045$	0	0

Here, we can see that once the scientists learn Neptune exist, they then become highly confident that Newtonian mechanics is true, even more so than their initial credences since $P_p(h_N) = 0.9 < P_a(h_N) = 0.96$. This delivers intuitive results.

However, the orthodox Bayesian updating tools of conditionalization delivered these results in part because of careful construction and updating. The prior distribution was deliberately constructed to deliver the results (in writing this paper, it took a few attempts to engineer the prior distribution to get results like this). Furthermore, it also took some time to perform the necessary calculation to update the results.

In contrast, the theorem of successful accommodation requires less effort to see whether one shot updates make ad hoc appeals to auxiliary hypotheses. One can simply plug the values into the inequality $P(e|h_2) \leq P(e|h_1\&a)P(a|h_1)$ to determine whether $\frac{P(h_1)}{P(h_2)} \leq \frac{P(h_1\&a|e)}{P(h_2|e)}$ and consequently whether the auxiliary hypothesis is ad hoc. Provided that the values that are plugged in are sensible, there is no need need for an exhaustive and careful assignment of probabilities to the space of possible outcomes which are then updated.

The merit of the formalism, then, is that it provides a simple and illuminating account of ad hoc hypotheses for one-shot updates, even though it does not assist with sequential updating.

This formalism, then, can also be extended to illuminate other concepts in the philosophy of science.

3. A Bayesian Account of Consilience

So far, we have seen that attempts to accommodate evidence come at a trade-off, and we have specified a condition under which those attempts are successful—at least according to one account. As it turns out, however, this also has implications for another topic in the philosophy of science: the virtue of *consilience* or, as it's sometimes called, *unification*.

In its Latin roots, "consilience" literally means "jumping together", and it was introduced by William Whewell—the person who also coined the term "scientist".[8] Whewell used the term "consilience" to refer when multiple chains of reasoning lead us to the same conclusion. Whewell further claims that "This Consilience is a test of the truth of the Theory in which it occurs."[9] Other philosophers of science have also

[8] Whewell is also recognized as the first systematic writer on the nature and history of science, and he coined other terms like "physicist". Yeo, *Defining Science : William Whewell, Natural Knowledge, and Public Debate in Early Victorian Britain*.

[9] *The Philosophy of the Inductive Sciences founded upon their History* (2d ed., London: John W. Parker, 1847), 469,.Quoted in Larry Laudan, "William Whewell on the Consilience of Inductions," ed. Sherwood J. B. Sugden, *Monist* 55, no. 3 (August 1, 1971): 369, https://doi.org/10.5840/monist197155318.

endorsed the value of consilient or unifying explanations, including Carl Hempel, Clark Glymour, Branden Fitelson and Thomas Blanchard.[10] More recently, Robert Nola has construed the term to refer to when a hypothesis explains multiple classes of fact.[11] We will adopt Nola's construal here. (Note that others, notably Stathis Psillos, use the term *consilience* differently.)[12]

The task of this section is to give a Bayesian account of the virtue of consilience and to relate that to the topic of ad hoc auxiliary hypotheses.

I will next describe the account in general terms and I will then illustrate it using the example of evolutionary theory.

3.1. Successful Consilience

Often, for any unifying explanation of some phenomena, there is some non-unifying alternative explanation. For example, you could often explain several health symptoms in terms of one disease which causes all of them, or you could explain each symptom as being caused by a separate ailment. Typically, then, some unifying explanation h_1 then competes with a set of alternative hypotheses. Maybe one of these alternative hypotheses is the main alternative hypothesis, while the others are auxiliaries that attempt to accommodate the evidence. Or maybe all of these alternative hypotheses are in some sense the main hypotheses.

In any case, the below account explicitly connects the success of a unifying explanation to the probability of the alternative hypotheses. And it does this in a way which closely resembles the account of successful accommodation above.

More generally, then, let h_1 and h_2 be competing hypotheses, let $e_1 \& \ldots \& e_n$ be a conjunctive statement denoting *n* items of evidence, and let $a_1 \& \ldots \& a_m$ be the conjunction of $m \leq n$ main or auxiliary hypotheses intended to accommodate the evidence when conjoined with h_2. Then, we can say that:

Account of Successful Consilience:

(4) h_1 successfully *consiliates* the evidence $e_1 \& \ldots \& e_n$ relative to a set of alternative hypotheses $\{h_2, a_1, \ldots, a_m\}$ just in case:

$$\frac{P(h_1)}{P(h_2)} < \frac{P(h_1 | e_1 \& \ldots \& e_n)}{P(h_2 \;\&\; a_1 \& \ldots \& a_m | e_1 \& \ldots \& e_n)}$$

[10] Clark Glymour, *Theory and Evidence* (Princeton: Princeton University Press, 1980); C G Hempel, *Philosophy of Natural Science* (Prentice-Hall, 1966); Branden Fitelson, "A Bayesian Account of Independent Evidence with Applications," *Philosophy of Science* 68, no. 3 SUPPL. (October 22, 2001), https://doi.org/10.1086/392903; Elisabet Tubau, Diego Alonso, and Diego Alonso, "Overcoming Illusory Inferences in a Probabilistic Counterintuitive Problem: The Role of Explicit Representations," *Memory & Cognition* 31, no. 4 (June 2003): 596–607, https://doi.org/10.3758/BF03196100; Blanchard, "Bayesianism and Explanatory Unification: A Compatibilist Account."

[11] Nola, "Darwin's Arguments in Favour of Natural Selection and Against Special Creationism." Some might also use the term "unification" to refer to this same phenomenon, but the term is also used in other ways which are very different from that of consilience. Todd Jones, "Unification," in *The Routledge Companion to Philosophy of Science*, ed. Martin Curd and Stathis Psillos, 2nd editio (Oxford; New York: Routledge, 2014), 552–60.

[12] Psillos uses consilience to refer to when an explanation fits with background knowledge. Stathis Psillos, "Simply the Best: A Case for Abduction," in *Computational Logic: Logic Programming and Beyond* , vol. 2408 (Springer Verlag, 2002), 605–25, https://doi.org/10.1007/3-540-45632-5_24.

What this says, then, is that h_1 successfully consiliates the evidence relative to some alternative h_2 just in case h_2 & a_1& … & a_n cannot successfully accommodate the evidence.

And it turns out that this condition is satisfied whenever h_2 & a_1& … & a_n fails to successfully accommodate the evidence à la the theorem of successful accommodation. We can then extend this to articulate the following theorem:

Theorem of Successful Consilience:

$$(5)\ \frac{P(h_1)}{P(h_2)} < \frac{P(h_1|e_1\& \ldots \& e_n)}{P(h_2\ \&\ a_1\& \ldots \& a_m|e_1\& \ldots \& e_n)}$$

iff

$$P\big(e_1\& \ldots \&e_n \,\big|\, h_2\&a_1\& \ldots \& a_n\big)P(a_1|a_2\& \ldots \&a_m\&h_2)\ldots P(a_n|h_2\,) < P(e_1\& \ldots \& e_n|h_1)$$

The proof of this is again found in the appendix.

Let us illustrate this more concretely, using the example of Darwin's observations in favor of evolution.

3.2. Illustrative Application: Darwin's Observations in Favor of Evolution

But before we get into the details, a disclaimer is in order: the purpose of this section is not to reconstruct how Darwin *actually did* reason, but rather to provide a Bayesian analysis of how one *could have* reasoned in order to reach Darwin's same conclusion in favor of evolution.

With that in mind, let us examine his argument in more detail.

Darwin was largely arguing against *special creationism*, the hypothesis that a God created each species in a separate creative act. According to special creationism, God created the chickens in one act, the lizards at another, and the bonobos at another. On this picture, it is not as though God created one species and that species then evolved into another.

Note, however, that Darwin did not present his argument as one against the religion or the existence of God in general. Indeed, writing in the 6th edition of the *Origin of Species,* he states quite clearly:

> I see no good reasons why the views given in this volume should shock the religious feelings of any one…. A celebrated author and divine has written to me that 'he has gradually learned to see that it is just as noble a conception of the Deity to believe that He created a few original forms capable of self development into other and needful forms, as to believe that He required a fresh act of creation to supply the voids caused by the action of His laws.'[13]

It then seems that Darwin took himself to be arguing not against religion in general, but rather against a *specific religious view*—special creationism—that was held by some followers of religion, but not all of them.

His preferred alternative hypothesis was that the species evolved by natural selection. According to this hypothesis, species vary in their biological features, and some variations are more conducive to survival in their environmental circumstances. Darwin believed, for example, that birds with webbed feet had an advantage over those without webbed feet in aquatic environments. Consequently, species with advantageous features were more likely to survive, and this forms the basis for "natural selection". Such processes of variation continue, and consequently some species give rise to other species. For example, some species of non-aquatic birds that live of land then evolved from aquatic birds with webbed feet, and

[13] Charles Darwin, *On The Origin of Species*, 6th ed. (Feedbooks, 1872).

this is why those non-aquatic birds have webbed feet even though they live on land. All species, then, ultimately evolved via this process of variation and natural selection and, for this reason, many species shared common ancestors (although it is not currently clear to me whether Darwin believed *all* species descended from the *same* ancestor or several of them). On this view, humans and other primates evolved from a common ancestor, and that explains why humans have tailbones that are "useless in humans".

This view strongly contrasted with special creationism. On Darwinian evolutionary theory, humans and other primates evolved through variation and natural selection from a *common* ancestor. On Special Creationism, humans and other primates were made by God in *separate* creative acts—and not by descent from a common ancestor. The conclusion of Darwin's argument is that evolutionary theory is a better explanation of various phenomena than special creationism.

What, then, are the phenomena which evolutionary theory sought to explain?

Well, there are many. Some of these are concisely discussed by Nola, and I summarize these below.[14]

1. The Dissimilarity between Blind Insects in American and European Caves

Darwin notes that blind insects in the caves of America and Europe have similar environments, and so we might have expected similarities in their biology if God created these insects for the similar environments, but separately from the insects outside of the caves. However, he notes that this is not the case. The blind insects in the American caves are quite dissimilar from those inside the European caves. Furthermore, the blind insects in the American caves more closely resemble the non-blind insects in America outside of those caves, and a similar point holds for the European insects. He believes that the better explanation for this is that the blind insects inside the American caves descended from ancestors immediately outside of those American caves, and similarly for the European insects. This, Darwin thinks, favors natural selection over special creationism.

2. Web-Footed, Aquatic Birds

Darwin notes that if each creature is "created as we now see it", as special creationists believe, then it is surprising when animals occasionally have biological structures that do not match their habits. He mentions numerous examples of this. One of them is that some geese living on dry land seldom, if ever, go near the water. Despite that, these geese have webbed feet. There is then a mismatch between their structure and their habits, between the fact that they have webbed feet on the one hand and the fact that they rarely if ever go near water on the other.

He thinks this is quite probable given evolution by natural selection, for such geese may still have evolutionary advantages in the non-aquatic habitats they dwell in. However, he thinks special creationism does not offer a satisfactory explanation:

> He who believes in separate and innumerable acts of creation will say, that in these cases it has pleased the Creator to cause a being of one type to take the place of one of another type; but this seems to me only restating the fact in dignified language.

One might think—as Nola does—that Darwin here is making a stronger claim: that special creationism offers *no* explanation here at all.[15] In any case, he clearly prefers the explanation offered by evolution by natural selection.

[14] Nola, "Darwin's Arguments in Favour of Natural Selection and Against Special Creationism."
[15] Nola.

3. No Leaps in Structure

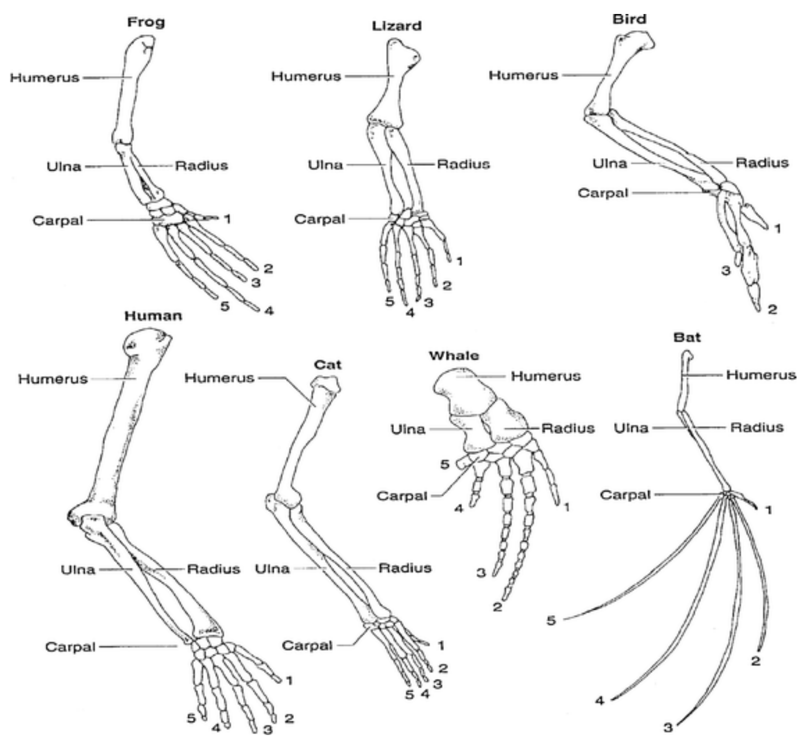
Darwin notes that the organs of species often appear to be related to earlier transitional organs. The evolutionary explanation of this is that such biological structures evolved from each other by “the shortest and slowest steps”. Hence, we would expect many existing species to be predated by earlier transitional forms. However, he asks why, if special creationism is true, “should not Nature take a sudden leap from structure to structure?” Again, he prefers the explanation offered by evolutionary theory.

4. Similarities in Morphology

In biology, a “homology” is a technical term that refers to the similarity between a pair of structures due to the fact that the structures both descended from a common ancestor. Darwin discusses an example of a homology: the shared bone structure between species. He states:

> What can be more curious than that the hand of a man, formed for grasping, that of a mole for digging, the leg of the horse, the paddle of the porpoise, and the wing of the bat, should all be constructed on the same pattern, and should include the same bones, in the same relative positions?

The below graph, reproduced from Nola, illustrates this phenomenon across various species:



We can then see numerous homologies between these species: the fact they all share one humerus bone preceding an ulna and a radius; the fact that (most of) these species share five digits or fingers, and so forth. He claims that the explanation of this is clear given evolution by natural selection: these similarities in structure exist because the species evolved by gradual steps from a common ancestor. Again, however, Darwin expresses dissatisfaction with the special creationist alternative:

> On the ordinary view of the independent creation of each being, we can only say that so it is;—that it has so pleased the Creator to construct each animal and plant.

3.3. *Darwin's Inference in Favor of Evolution*

So those are few of the many phenomena that Darwin appeals to in support of evolution by natural selection.

How, more specifically, does he then infer that evolutionary theory is the preferable explanation?

Well, there are a couple of interpretations available here.

One is the claim that evolutionary theory offers an explanation of observations whereas special creationism offers no explanation at all. As Darwin says in one case, special creationism may appeal to the possibility that it just "so pleased the Creator to construct each animal and plant", but he complains in another case that this "seems to me only restating the fact in dignified language". Some might interpret this as saying that evolution offers an explanation whereas special creationism does not.

Another interpretation is that regardless of whether special creationism offers an explanation, evolutionary theory nevertheless offers a *conciliatory* explanation: it explains diverse classes of fact in a way that is so satisfactory that it cannot be false. This interpretation appears to be clearly affirmed by Darwin himself:

> It can hardly be supposed that a false theory would explain, in so satisfactory a manner as does the theory of natural selection, the several large classes of facts above specified. It has recently been objected that this is an unsafe method of arguing; but it is a method used in judging of the common events of life, and has often been used by the greatest natural philosophers. The undulatory [wave] theory of light has thus been arrived at; and the belief of the revolution of the Earth on its own axis was until lately supported by hardly any direct evidence.

Here, he expresses incredulity that a "false theory" could explain the above phenomena and others in "so satisfactory a manner". He further thinks that a similar "method of arguing" is used in both the "common events of life" as well as in natural philosophy—where "natural philosophy" refers to what we now call "science". And he mentions two success stories in science as support of this: the wave theory of light and the universal law of gravitation, both of which are discussed by Whewell. (In fact, Nola argues that Whewell actually inspired Darwin's conception of science and the scientific method here.)

So Darwin gives us with some insight into his reasoning process.[16]

However, we are less interested here in explaining how Darwin actually did reason and more how one *could have* reasoned in a Bayesian framework. This, then, is the topic of the next subsection.

3.4. *A Bayesian Analysis of Consilience and Evolutionary Theory*

Let us frame the debate about evolution in probabilistic terms.

More specifically, let h_{NS} be the Darwinian hypothesis that the species evolved by natural selection from a common ancestor, let h_{SC} be the hypothesis that every species was created separately and as is by God. Then, let the following symbols denote the different pieces of evidence above:

> e_{BI} = the blind insects in the American caves are quite dissimilar from those inside the European caves, and the blind insects in the two continents more closely resemble the non-cave dwelling insects outside their respective caves

[16] This example also played an important role in Kitcher's early account of scientific explanation as unification. Kitcher, "Explanatory Unification."

e_{WB} = some birds have webbed feet but are non-aquatic

e_{LS} = there are no leaps in structure between the species, and instead many biological structures appear to be predated by earlier transitional forms

e_H = the species share many homologies, including the common bone structure in the arms of humans, horses, moles, bats and other animals.

These pieces of evidence together constitute some of the phenomena that Darwin appeals to in his argument for evolution. Let us denote these pieces of evidence with the conjunctive statement e_{BI} & e_{WB} & e_{LS} & e_H& ....

We can then use this notation to reason about how the evidence affects the probability of the main hypotheses h_{NS} and h_{SC}. Darwin appears to think that the probability of the evidence is high on evolution by natural selection. After all, if the species evolved by natural selection, then we would not be surprised to see that blind insects in the American caves are quite dissimilar from those inside the European caves, that some birds have webbed feet but are non-aquatic, and so on and so forth. For illustrative purposes, then, we might say that $P(e_{BI} \,\&\, e_{WB} \,\&\, e_{LS} \,\&\, e_H \,\&\, ... | h_{NS}) = 0.7$. But again, in this context, what matters is not so much the specific value but more so that it is higher than other quantities, including the likelihood given the alternative hypotheses.

So, then, how likely is the evidence given special creationism?

Well, it seems not very likely. After all, the mere hypothesis that God created every species separately does not by itself lead us to expect that blind insects in the American caves are quite dissimilar from those inside the European caves, that some non-aquatic birds have webbed feet and so on.

Darwin then anticipates some auxiliary hypotheses which the special creationist might offer. In discussing the webbed feet of non-aquatic birds, he says: "He who believes in separate and innumerable acts of creation will say, that in these cases it has pleased the Creator to cause a being of one type to take the place of one of another type". This paves the way to a generic response to the evidence which a creationist might give: for any fact about the species, the special creationist could say that it simply "pleased" God to create the species in that way. In our notation, then, let the auxiliary hypothesis a_{NL} denote the proposition that God simply desired to make it so that there are no leaps in structure between the species, let the auxiliary hypothesis a_H denote the proposition that God simply desired to make it so that the species share many homologies, and so on for the other pieces of evidence.

Then, the special creationist might again claim that even though $P(e_{BI} \,\&\, e_{WB} \,\&\, e_{LS} \,\&\, e_H \,\&\, ... | h_{SC})$ appears low, $P(e_{BI} \,\&\, e_{WB} \,\&\, e_{LS} \,\&\, e_H \,\&\, ... | h_{SC} \,\&\, a_{BI} \,\&\, a_{WB} \,\&\, a_{LS} \,\&\, a_H \,\&\, ...)$ is much higher.

Again, we see an attempt to accommodate the evidence by appealing to auxiliary hypotheses to raise the likelihood of the evidence. And in this case, clearly something seems suspicious about this attempt.

What is it then?

Darwin offers a critique, but it is not very detailed. For example, he discusses the creationist explanation that it simply "pleased the Creator" to make aquatic birds "take the place" of the non-aquatic birds. What is his complaint about this? He says nothing more than this: "this seems to me only restating the fact in dignified language". It seems, then, that his criticism is that it does not really explain the fact in question, but it seems to be a mere restatement of it.

Of course, we might not be satisfied with Darwin's complaint—rightly or wrongly. To state that some mainland birds have webbed feet seems quite distinct from stating that *God desired there to be mainland birds with webbed feet*. One statement seems distinct from the other, and so it is not exactly clear how one is merely a restatement of the other. Perhaps, however, there is some more compelling or charitable interpretation of Darwin here: one might think that he is somehow getting at the idea that the auxiliary hypothesis is not supported by independent evidence, or something to that effect.

In any case, as mentioned above, the point is not to describe and evaluate what Darwin thought.

Instead, it is to provide a formal Bayesian account of how one *could* reason about this case. So, then, is there a different criticism of these auxiliary hypotheses?

Here is my suggestion: evolution by natural selection successfully consiliates the evidence, and the auxiliary hypotheses are ad hoc!

To see this, let us apply the above account of successful consilience to our example.

According to the account, evolution by natural selection h_{NS} successfully *consiliates* the evidence e_{BI} &e_{WB}& ... relative to a set of alternative hypotheses $\{h_{SC}, a_{BI}$ & a_{WB}& ...$\}$ just in case:

$$\frac{P(h_{NS})}{P(h_{SC})} < \frac{P(h_{NS}|e_{BI} \,\&\, e_{WB} \& \dots)}{P(h_{SC} \,\&\, a_{BI} \,\&\, a_{WB} \& \dots|e_{BI} \,\&\, e_{WB} \& \dots)}$$

What this says, then, is that h_{NS} successfully consiliates the evidence relative to special creationism h_{SC} just in case special creationism cannot successfully accommodate the evidence with its auxiliaries.

And according to the theorem of successful consilience, it will be successful as such just in case:

$$P\big(e_{BI} \,\&e_{WB} \& \dots \,\big|\, h_{SC} \,\&\, a_{BI} \,\&\, a_{WB} \,\&\, \dots\big)P\big(a_{BI}\,\big|\, a_{WB} \,\&\, a_{LS} \ \dots \&h_{SC}\big) \dots P(a_i|h_{SC}) \\ < P(e_{BI} \,\&\, e_{WB} \& \dots |h_{NS})$$

(where a_i is the final auxiliary hypothesis that helps special creationism accommodate the evidence)

Crucially, then, to determine whether special creationism successfully accommodates the evidence, we must consider the probability of each of the auxiliary hypotheses. And on the Bayesian analysis, this is the locus of the problem in special creationist reasoning: each auxiliary hypothesis is less than certain, and consequently the appeals to such hypotheses progressively compromise the ability of special creationism to accommodate the evidence.

For illustrative purposes, let us assign values to the probability of each auxiliary hypothesis. Suppose that prior to learning of the evidence, we had no strong reason to expect God to want the evidence that way. For example, prior to learning about there being no leaps in structures, suppose we have no reason to expect God to make it so that there were no leaps in structures, and similarly for the other items of evidence. For that reason, then, we can suppose $P(a_{LS}) = 0.5$, and similarly for the other evidence about the dissimilarity between insects in different caves, the non-aquatic birds with webbed feet and so forth. And furthermore, let us suppose for illustrative purposes that all of these auxiliary hypotheses are probabilistically independent: for instance, learning that God wants non-aquatic birds to have webbed feet would not affect the extent to which we would expect insects in caves in Europe and America to be different. Put formally, then, let $P(a_i) = P(a_i|a_1\& \dots \&a_k)$ for any auxiliary hypotheses a_i and a_j such that $i \neq j$ for any $1 \leq j \leq k$ and where $1 \leq k$.

Using the above values, we can then show formally both that evolutionary theory successfully consiliates the evidence while special creationism and its auxiliaries fail to successfully accommodate it.

$$P\big(e_{BI} \,\&e_{WB} \& \dots \,\big|\, h_{SC} \,\&\, a_{BI} \,\&\, a_{WB} \,\&\, \dots\big)P\big(a_{BI}\,\big|\, a_{WB} \,\&\, a_{LS} \ \dots \&h_{SC}\big) \dots P(a_i|h_{SC}) \\ = (1)(0.5)(0.5)(0.5)(0.5) \dots \\ < P(e_{BI} \,\&\, e_{WB} \,\&\, e_{LS} \,\&\, e_H \& \dots |h_{NS}) = 0.7$$

As we can see, the product of the left of the inequality is less than 0.0625, while the value on the right hand of the inequality is 0.7. By implication, then, Darwinian evolutionary theory does a great job of conciliating the evidence while special creationism and its auxiliaries offer only an ad hoc attempt to accommodate the evidence. This, to my mind, represents a satisfactory analysis of how one *could* have reasoned about evolution and creationism in Bayesian terms.

But again, it does so only given the earlier caveats about one-shot vs. sequential updating: the theorem of successful consilience illuminates how one *could* reason about the evidence in a case of one-shot updating, but not in a case of sequential updating.

4. Implications and Relevance to the Literature

In the preceding sections, I have outlined a Bayesian account of when hypotheses successfully accommodate counter-evidence, when they are ad hoc and when they successfully consiliate disparate evidence. The task of this section is to explain how these accounts relate to extant literature—more specifically, by outlining how they resemble or differ from extant accounts.

4.1. Other Accounts of Auxiliary Hypotheses and Ad Hocness

Philosophers have given various accounts of ad hocness and similar concepts. There are too many to discuss here, but Samuel Schindler provides a helpful overview of the main accounts elsewhere.[17]

These other accounts often differ in that they do not provide *quantitative* analyses of ad hocness. Instead, ad hocness is described in terms of qualitative criteria. According to Karl Popper, for example, an ad hoc hypothesis is one which is not independently testable—that is, if it is true, it does not lead us to expect other consequences that can be independently tested.[18] This account is purely qualitative: the ad hocness of a hypothesis is assessed simply by whether it meets this criterion of independent testability. One might think the account in this paper is also qualitative in one sense since it is comparative. But regardless, it is surely "quantitative" in another sense: it incorporates some specific quantities—probabilities—that determine whether an auxiliary hypothesis is ad hoc, and it is surely quantitative in this sense, even if it is not quantitative in others.

Of course, there are some Bayesian philosophers of science who discuss auxiliary hypotheses or similar accounts, but none of them thoroughly articulate a quantitative account of ad hocness. For example, Jon Dorling provides an excellent discussion of how evidence bears on the probabilities of both main and auxiliary hypotheses, yet he does not use this to determine which auxiliary hypotheses are *ad hoc*.[19] One might have expected a characterization of ad hocness in Colin Howson and Peter Urbach's pioneering work *Scientific Reasoning: the Bayesian Approach.* Instead, however, all one finds is a criticism of some qualitative accounts of ad hocness. They do claim, however, that the Bayesian approach can explain why some react with "incredulity" when "certain ad hoc hypotheses are advanced"—namely, because they are "struck by the utter implausibility" of those hypotheses.[20] I do not disagree with the truth of this claim; rather, the point of this paper is to integrate its truth into a formal account of ad hocness. Despite this, however, Howson and Urbach do not themselves offer a formal litmus test for ad hocness.

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[17] A review of various accounts of ad hocness can be found in Samuel Schindler, *Theoretical Virtues in Science* (Cambridge: Cambridge University Press, 2018), chap. 5.

[18] Karl Raimund, Popper, *Objective Knowledge; an Evolutionary Approach* (Oxford, Clarendon Press, 1972); Greg Bamford, "Popper's Explications of Ad Hocness: Circularity, Empirical Content, and Scientific Practice," *British Journal for the Philosophy of Science* (Oxford Academic, June 1, 1993), https://doi.org/10.1093/bjps/44.2.335.

[19] Jon Dorling, "Bayesian Personalism, the Methodology of Scientific Research Programmes, and Duhem's Problem," *Studies in History and Philosophy of Science Part A* 10, no. 3 (September 1979): 177–87, https://doi.org/10.1016/0039-3681(79)90006-2.

[20] Colin Howson and Peter Urbach, *Scientific Reasoning: The Bayesian Approach*, 3rd ed (Chicago: Open Court, 2006), 124.

The closest one finds to a formal discussion of ad hocness in the literature is Michael Strevens' illuminating discussion of auxiliary hypotheses in response to the Quine-Duhem problem.[21] There, he discusses when a "glorious rescue" of a main hypothesis occurs:

> In words, a glorious rescue occurs roughly when the auxiliary hypothesis receives most of the blame for a false prediction, and is rightly discarded by researchers in favor of some other auxiliary hypothesis that makes the correct prediction. (The degree of glory, I remark in passing, is perhaps inversely proportional to the prior probability of the ad hoc hypothesis.)

In this context, he provides an insightful account of a response to the Quine-Duhem problem. But as one can see, then, he makes a mere parenthetical and speculative remark about ad hoc hypotheses.

In this paper, however, ad hocness takes on a more central focus.

Despite this, though, this paper does not offer an account of ad hocness that *competes* with Strevens' account. If anything, it aims merely to *complete* and *complement* his account. It does this by offering a new formalism by which to assess ad hocness (the theorem of successful accommodation in this paper) and by relating that formalism to the topic of consilience. None of these things are obvious or entailed from Strevens' parenthetical comment above—which is all he says on the matter. Consequently, it offers an account to complement Strevens' discussion on the topic.

4.2. Others Analyses of Evolution

This paper's analysis of ad hocness and evolution also differs from analyses offered by others. Various philosophers have discussed why special creationism is defective, and there are several candidate answers.[22]

Sober, for example, claims that the problem with the special creationist hypothesis is that it tries to accommodate the data by appealing to auxiliary hypotheses that have no independent justification (a criticism that is similar to what Popper would say).[23] For example, while evolution could explain morphological similarities between species by appealing to evolution from a common ancestor, special creationism appeals to an otherwise unsupported hypotheses to explain the same evidence: namely, that it simply pleased God to create the species with such similarities. Sober then complains that the special creationist fails to offer independent justification for their hypotheses about the goals and abilities of the God.

Maarten Boudry and Bert Leuridan criticize Sober's view, claiming that sometimes appeals to auxiliary hypotheses are legitimate even if they lack independent justification.[24] Instead, they join Philip Kitcher and (seemingly) Darwin in claiming that the problem with intelligent design is that it lacks the virtue of

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[21] Strevens does, however, make some remarks about "desparate rescues" of a main hypothesis. However, this is distinct from an ad hoc auxiliary hypotheses since such rescues occur when a "central hypothesis receives most of the blame for a false prediction" but researchers "discard the evidently superior auxiliary". In other words, this occurs when the auxiliary hypothesis is a good one and the main . Michael Strevens, "The Bayesian Treatment of Auxiliary Hypotheses," *British Journal for the Philosophy of Science* 52, no. 3 (September 1, 2001): 515–37, https://doi.org/10.1093/bjps/52.3.515.

[22] Kitcher, "Explanatory Unification"; Elliott Sober, *Evidence and Evolution : The Logic behind the Science* (Cambridge University Press, 2008); Maarten Boudry and Bert Leuridan, "Where the Design Argument Goes Wrong: Auxiliary Assumptions and Unification*," *Philosophy of Science,* vol. 78, 2011, http://www.journals.uchicago.edu/t-and-c; Nola, "Darwin's Arguments in Favour of Natural Selection and Against Special Creationism."

[23] Sober, *Evidence and Evolution : The Logic behind the Science*.

[24] Boudry and Leuridan, "Where the Design Argument Goes Wrong: Auxiliary Assumptions and Unification*."

consilience (although Kitcher, Boudry and Leuridan instead use the term *explanatory unification* to refer to what is essentially this same virtue).[25]

The theorem of successful consilience may to some extent jibe with the perspectives of all of these philosophers. According to the above analysis, evolutionary theory is a successful theory because it successfully consiliates the evidence. We can agree with Darwin, Kitcher, Boudry and Leuridan on this much. And in contrast, special creationism falters as it requires numerous ad hoc auxiliary hypotheses—and consequently numerous costly trade-offs—to accommodate the same evidence. In this case, each auxiliary hypothesis has a low probability prior to receiving the evidence: for example what reason would we have for expecting God to create various similarities between species *prior to us knowing the fact of their similarity?* Of course, if we had independent justifications to believe this, then the prior probability would be higher. For that reason, we can agree with Sober to the extent that the auxiliary hypotheses would have been more appropriate if they had such justifications. But lacking such justifications, these auxiliaries come at a cost which compromises the ease with which special creationism can accommodate the evidence. Ultimately, though, the locus of the problem is the probability of the auxiliary hypotheses, and justifications matter only in virtue of this.

4.3. Ad Hocness in Other Philosophical Debates

Aside from evolution, the accounts in this paper may also have relevance to other areas. Consider, for example, Robin Collins' discussion of fine-tuning in the philosophy of religion.[26] Here, the main datum is that the physical laws and constants of the universe seem finely-tuned so as to permit life. Supposedly this datum supports theism—that is, in this context, the hypothesis that God exists. Fine-tuning arguments typically assert that such fine-tuning is more likely given theism (which we can denote with h_T) than given a single atheistic universe (which we can denote with h_A). For that reason, fine-tuning supports theism—or so Collins' argument goes. However, Collins is also aware that the evidence fine-tuning is not so unlikely given a reformulated version of the atheistic hypothesis. The reformulation is basically h_A conjoined to the proposition *a* that our universe is life-permitting. And one might be sympathetic to this reformulation: after all, the universe would need to have values tuned so as to permit life if it was indeed life permitting, so fine-tuning would not be surprising given this hypothesis. But Collins claims there is something fishy about this reformulation—namely, that it suffers from what he calls *probabilistic tension*. Using our notation (not his), Collins would say that probabilistic tension occurs when a conjunct in the conjunction h_A & a is "very unlikely" when conditioned on the other conjunct.[27] Now he claims that the reformulated hypothesis suffers from such tension because it is very unlikely that the universe would be life permitting given a single atheistic universe. He then provides examples of probabilistic tension in other cases, and he uses these to intuitively motivate the idea that probabilistic tension provides a reason to "reject" a hypothesis.[28]

Now my point here is not to endorse or reject Collins' argument, especially when I have not discussed other objections or alternative explanations (like the so-called "multiverse" alternative explanation of fine-tuning which I think is much better than h_A). Rather, my aim is to point out that the spirit of his account of probabilistic tension is captured by the theorem of successful accommodation. On this account, an auxiliary hypothesis successfully accommodates the evidence only if $P(e|h_T) \leq P(e \mid h_A \,\&\, a)P(a|h_A)$. But if the auxiliary hypothesis is very improbable given the main hypothesis—in other words, if $P(a|h_A)$ is very

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[25] Kitcher, "Explanatory Unification"; Boudry and Leuridan, "Where the Design Argument Goes Wrong: Auxiliary Assumptions and Unification"; Nola, "Darwin's Arguments in Favour of Natural Selection and Against Special Creationism."

[26] Robin Collins, "The Teleological Argument: An Exploration of the Fine-Tuning of the Universe," in *The Blackwell Companion to Natural Theology*, ed. William Lane Craig and J.P. Moreland (Malden: Blackwell Publishing Inc, 2009), 202–81.

[27] Collins, 209.

[28] Ibid.

low—then in Bayesian terms, *this* is the reason why appealing to the auxiliary hypothesis is blameworthy. In this sense, rightly or wrongly, Collins might say that the reformulated atheistic hypothesis is ad hoc because the probability of the auxiliary hypothesis is very low given the hypothesis—that is, the probability of the universe permitting life is very low if we live in a single atheistic universe.

So the theorem captures the spirit of Collins' proposal, but it does so for a different reason—a stronger one, in my opinion. Collins motivates his principle with examples and intuition. In contrast, our theorem follows from axioms that are widely endorsed by Bayesians and by probability aficionados more generally. But of course, the theorem of successful accommodation also is broader than Collins constraint. Unlike Collins' account, it exhorts us to be sensitive to relevant parameters: for example, even if the prior probability of the auxiliary hypothesis is low, it may still successfully accommodate the evidence if the likelihood of the evidence given the other competing hypothesis is that much lower.

In any case, the point is not to criticize Collins' constraint, but just to provide a tangible example of how the theorem may be useful to debates in fields aside from the philosophy of science.

4.4. *Other Accounts of Consilience*

In comparison to ad hocness, consilience has received more explicit attention in the Bayesian literature. Perhaps the most recent treatment of the topic comes from Thomas Blanchard.[29] Blanchard aims to show that the Bayesian framework naturally gives weight to the virtue of consilience (a virtue which he calls "explanatory unification").

To do this, he appeals to the notion of "mutual information" or *MI*, as discussed by Wayne Myrvold. According to Myrvold, a hypothesis is *MI-unifying* when it "render[s] what, on prior grounds, appear to be independent phenomena informationally relevant to each other".[30] The function $MIU(.)$ is a measure of how MI-unifying a hypothesis h is, and it is defined as such for *n* items of evidence and some credence function *C*:

$$MIU(e_1, \dots, e_n; h) = \log_2\left[\frac{C(e_1\& \dots \&e_n|h)}{C(e_1|h)\dots C(e_n|h)}\right] - \log_2\left[\frac{C(e_1\& \dots \&e_n)}{C(e_1)\dots C(e_n)}\right]$$

Put simply, this measures the extent to which acceptance of the hypothesis would increase how much one expects *all* the evidence *as a whole*. If a hypothesis is MI-unifying in this sense, then accepting it would increase our expectation of all the evidence which it MI-unifies. The probability of a hypothesis given the evidence can be specified in terms of MI-unification. In particular, for two competing hypotheses h_1 and h_2, their respective posterior probabilities are related to the $MI(.)$ measure via the following equation:

$$\begin{aligned}\log_2\left[\frac{C(h_1 \mid e_1\& \dots \&e_n)}{C(h_2 \mid e_1\& \dots \&e_n)}\right] \\ = \log_2\left[\frac{C(e_1 \mid h_1)}{C(e_1 \mid h_2)}\right] + \dots + \log_2\left[\frac{C(e_n \mid h_1)}{C(e_n \mid h_2)}\right] \\ + \log_2\left[\frac{C(h_1)}{C(h_2)}\right] \\ + MIU(e_1, \dots, e_n; h_1) - MIU(e_1, \dots, e_n; h_2)\end{aligned}$$

The value of this account is that is explicitly specifies the relationship between the posterior probability of some hypotheses on the one hand and the extent to which they MI-unify the evidence on the other. The

[29] Blanchard, "Bayesianism and Explanatory Unification: A Compatibilist Account."
[30] Wayne C Myrvold, "A Bayesian Account of the Virtue of Unification*," *Philosophy of Science*, vol. 70, 2003, 400.

account then, in my opinion, successfully shows how Bayesianism naturally gives weight to explanatory unification.

The account of consilience in this paper differs.

But, again, it is *not* a *competing* account: the account in this paper does not undermine the success of Blanchard and Myrvold's account.

Instead, it is simply a *complementary* account. Blanchard and Myrvold succeed in giving a Bayesian account of the important of consilience qua mutual information unification. The aim of this paper is different: to give a very simple Bayesian account of unification qua successful consilience, and one which explicitly relates the success of a conciliatory hypothesis to the ad hocness of competing auxiliary hypotheses. The theorem also explicitly highlights a way in which a hypothesis might *unsuccessfully* consiliate or unify the evidence: namely, when the evidence is sufficiently probable given alternative hypotheses with auxiliaries that are themselves sufficiently probable. I would hope that, like Blanchard and Myrvold, this paper succeeds in accomplishing its aims—aims that are, in this case, tangential but complementary to theirs.

5. Unresolved Questions

So we have outlined some theorems which elucidate the nature of successful accomodation, ad hocness and consilience. In this part, we now turn to discuss some final issues and outline some avenues of future research.

5.1. The Ubiquity of Auxiliary Hypotheses

This paper has worked with a picture of reasoning that might look over-simplified. In particular, our examples often describe scenarios where one hypothesis appeals to an auxiliary hypothesis while the other does not. In reality, however, *all* hypotheses are intimately linked to auxiliary hypotheses. The question then arises as to when such auxiliary hypotheses—and our uncertainty about them—are useful things to model in our formal representations. I suspect this will depend on the specific situation and that the relevant contexts and complexities are too numerous to enumerate here. However, this not the final answer, and so this is a question for future research.

5.2. The Problem of the Priors

We have considered a novel Bayesian approach to ad hocness and consilience. Perhaps a part of the reason such an account has not already been proposed is that it appeals to prior probabilities, at which point we run head-first into the problem of the priors: how do these prior probabilities get their values? This is also relevant to earlier questions about whether there are rational constraints on the assignment of priors in the case of the creationist: are we really to criticize the creationist when they may simply have different priors which can vindicate their views—even with the formalism? If the priors have no rational constraints,

A discussion of the Bayesian approach to ad hocness would then ideally feature a discussion of the problem of the priors. Of course, though, the problem of the prior deserves a book length treatment by itself and not merely the passing remarks in this paper.

I discuss the problem elsewhere in a working paper, arguing that in many contexts, we have empirical evidence to inform the relevant priors, and we can also criticize priors as being more or less trustworthy depending on whether we have reason to think they are produced by particular processes (namely, those that are “well-calibrated” and “maximally” inclusive in a sense I will not elaborate on here).[31] Consequently, I believe there is a good solution to the problem of the priors, but there is obviously no space to argue for this here when I do so in another paper. In any case, it should not be an especially objectionable problem for this paper given that a healthy amount of literature explores Bayesian treatments of various problems without tackling the problem of the priors.

5.3. Alternative Accounts of Accommodation

Earlier, I mentioned that there are alternative accounts of successful accommodation. We might then ask which, if any, is the best. I think this is a topic for future research, but I will share a few thoughts here, at the very least because one might object to my discussion unless I do so. In particular, one might think that a better account of accommodation is the following:

Alternative Account of Successful Accommodation:

(1) An auxiliary hypothesis a successfully accommodates the evidence with h_2 just in case:

$$\frac{P(h_2)}{P(h_1)} \leq \frac{P(h_2\&a|e)}{P(h_1\&a|e)}$$

And it turns out this will be true just in case the following condition holds.

Alternative Theorem of Successful Accommodation:

$$\frac{P(h_2)}{P(h_1)} \leq \frac{P(h_2\&a|e)}{P(h_1\&a|e)}$$

$$\text{iff}$$

$$P(e|h_1\&a)P(a|h_1) \leq P(e|h_2\&a)P(a|h_2)$$

One might think this account is better for this reason: if an auxiliary hypothesis a is to successfully accommodate the evidence with h_2, then the evidence should not still raise the probability of h_1 relative to h_2 when h_1 is also conjoined to that same auxiliary hypothesis.

This account may very well have some merit in some *particular* cases. However, I am not convinced it is better in *all* cases. To see this, consider the following scenario. Someone passes by your bedroom window; out the corner of your eye, you see some blue-ish color on their head. You are expecting two friends to come to your place today, and each of them have distinct hair colors. They will arrive separately, but you do not know when or in what order. One hypothesis h_1 is that the person who went by your window was Kayley, one of the two friends, and the friend with blue hair. The other hypothesis h_2 is that the person who went by your window was Kyle, the other friend, and the friend with brown hair. Let’s suppose you assigned an equal prior probabilities to either of them walking past your window, so $P(h_1) = P(h_2)$. You then consider your evidence e: that you saw someone with blue on their head. You think the evidence confirms h_1 since it is more likely that you would see blue if the person walking by your room had blue hair—as Kayley does—so $P(e|h_1) > P(e|h_2)$. But note that we could appeal to an auxiliary hypothesis a

[31]Note that some think of prior probabilities as being exclusively priors for some hypotheses prior to the receipt of *any evidence*. However, there is a broader notion of a prior probability which is a probability for some hypothesis prior to the receipt of *some evidence* (but not necessarily all evidence). It is this broader notion that I think is useful for this paper and that is more common in real-life situations. I discuss the assignment of priors in real-world contexts in John E. Wilcox, “Credences, Calibration and Trustworthiness,” 2021.

in an attempt to accommodate the evidence with h_2: the person walking by your room was wearing a blue hat. Actually, sometimes Kyle does wear a blue hat, but he does so very infrequently, so we can say $P(a|h_2) = .01$. On the other hand, Kayley hates hats, never wears them, and so we can suppose (for illustrative purposes at least) that $P(a|h_1) = 0$. Intuitively, this auxiliary hypothesis does not successfully accommodate the evidence with h_2: the evidence should still favor the hypothesis that Kayley walked by your window since she has blue hair and since it is very unlikely that Kyle wore a blue hat. But note that the above account fails to capture this because:

$$P(e|h_1\&a)P(a|h_1) = (1)(0) = 0 < .01 = (1)(.1) = P(e|h_2\&a)P(a|h_2)$$

So we have a result where intuitively the auxiliary hypothesis fails to successfully accommodate the evidence, but the above account incorrectly says otherwise. The problem here is that the relevant comparison is between $h_2\&a$ and h_1, not between $h_2\&a$ and $h_1\&a$: we want to compare the probability that Kyle walked by with a blue hat (a low probability event) and the probability that Kayley walked by (a moderate probability event)—but not the probability that Kayley walked by with a blue hat (a zero probability event). We might try to fix this example in various ways, but none of them work, from what I can see.[32] For that reason, the alternative account cannot be the correct account in all cases, even if it is for some of them.

Such considerations lead me to think that there may be multiple viable accounts of accommodation, but which is one is the best is *context-sensitive*: it depends on various features of the agent's context. What those features are, then, is a matter for future research. In any case, the accounts in this paper have their merits, and I suspect that the earlier "Account of Successful Accommodation" may suffice for many examples encountered in science and in everyday life.

6. Conclusion

In this paper, I have attempted to provide a formal Bayesian analysis of three concepts in the philosophy of science: successful accommodation, ad hocness and consilience. I have also applied this analysis to examine the reasoning which could have underpinned inferences in favor of evolution over special creationism. To my mind, the analysis represents a satisfactory and illuminating attempt to understand these concepts in ways which complement—but do not compete with—existing accounts on similar or related concepts.

Despite this, however, further work remains in understanding questions such as when auxiliary hypotheses are useful to model in our formal representations and when alternative accounts of accommodation are warranted.

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[32] For example, we might say that *a* should instead be the auxiliary hypothesis that *Kyle* was wearing a blue hat, not that the person walking by the window was wearing the blue hat. This does not help the alternative account, however. Recall that $P(a) = .01$. The prior probability of *a* is independent of h_1 and h_2, and so $P(a) = P(a|h_1) = P(a|h_2)$. Intuitively, the auxiliary does *not* successfully accommodate the evidence, since it raises the likelihood of the evidence by making a trade-off that is quite costly given the low prior probability of *a*. But according to the above account, it *does* successfully accommodate the evidence, all because:

$$P(e|h_1\&a)P(a|h_1) = (1)(.01) \le (1)(.01) = P(e|h_2\&a)P(a|h_2)$$

This is clearly the wrong outcome. $P(a|h_1)$ shouldn't matter: we are not interested in the probability that Kyle is wearing a blue hat given that Kayley walked by your window. But the account unreasonably takes this probability into account when considering whether the evidence supports h_1 over h_2. Hence, the alternative account cannot be correct in all cases.

Appendix: Proofs of Main Results

Likelihood-Prior Trade-Off principle:

(1) If $P(e|h) < P(e|h\&a)$, then $P(h) > P(h\&a)$ where $0 < P(h)$and $P(a|h) < 1$.

Proof: Suppose $P(h) > 0$ and $P(a|h) < 1$. Then, $P(h) = n$ for some $n \in (0,1]$ and $P(a|h) = m$ for some $m \in [0,1)$. Now by the probability calculus, $P(h\&a) = P(h)P(a|h) = nm$. Since *n* is a non-zero value, and *m* is less than 1, *nm* will necessarily be less than *n*, and so $P(h) > P(h\&a)$. This is true regardless of the values of $P(e|h)$ and $P(e|h\&a)$, so it is true in the special case where $P(e|h) < P(e|h\&a)$. Consequently, if $P(e|h) < P(e|h\&a)$, then $P(h) > P(h\&a)$ where $P(h) > 0$ and $P(a|h) < 1$.

Theorem of Successful Accommodation:

(3) $\frac{P(h_2)}{P(h_1)} \leq \frac{P(h_2\&a|e)}{P(h_1|e)}$ iff $P(e|h_1) \leq P(e|h_2\&a)P(a|h_2)$

Proof: By Bayes's theorem:

$$\frac{P(h_2)}{P(h_1)} \leq \frac{P(h_2\&a|e)}{P(h_1|e)} \text{ iff } \frac{P(h_2)}{P(h_1)} \leq \frac{\frac{P(e|h_2\&a)P(h_2\&a)}{P(e)}}{\frac{P(e|h_1)P(h_1)}{P(e)}}$$

We can then multiply both sides by $\frac{P(e)}{P(e)}$ (and since we are dealing with probabilities and none of the signs are negative, the inequality does not change):

$$\text{iff } \left(\frac{P(h_2)}{P(h_1)}\right)\left(\frac{P(e)}{P(e)}\right) \leq \left(\frac{\frac{P(e|h_2\&a)P(h_2\&a)}{P(e)}}{\frac{P(e|h_1)P(h_1)}{P(e)}}\right)\left(\frac{P(e)}{P(e)}\right)$$

We then have the following:

$$\text{iff } \left(\frac{P(h_2)}{P(h_1)}\right)(1) \leq \frac{P(e|h_2\&a)P(h_2\&a)}{P(e|h_1)P(h_1)}$$

By the probability calculus, we can express $P(h_2\&a)$ as $P(a|h_2)P(h_2)$:

$$\text{iff } \frac{P(h_2)}{P(h_1)} \leq \frac{P(e|h_2\&a)P(a|h_2)P(h_2)}{P(e|h_1)P(h_1)}$$

We can then derive the following:

$$\text{iff } \frac{P(h_2)}{P(h_1)} \leq \left(\frac{P(e|h_2\&a)P(a|h_2)}{P(e|h_1)}\right)\left(\frac{P(h_2)}{P(h_1)}\right)$$

Dividing both sides by $\frac{P(h_2)}{P(h_1)}$, we have the following:

$$\text{iff } \frac{\frac{P(h_2)}{P(h_1)}}{\frac{P(h_2)}{P(h_1)}} \leq \frac{\left(\frac{P(e|h_2\&a)P(a|h_2)}{P(e|h_1)}\right)\left(\frac{P(h_2)}{P(h_1)}\right)}{\frac{P(h_2)}{P(h_1)}}$$

We then have the following:

$$\text{iff } 1 \leq \frac{P(e|h_2\&a)P(a|h_2)}{P(e|h_1)}$$

And if we multiply both sides by $P(e|h_1)$, we then have our desired conclusion:

$$\text{iff } P(e|h_1) \leq P(e|h_2\&a)P(a|h_2)$$

Therefore, $\frac{P(h_2)}{P(h_1)} \leq \frac{P(h_2\&a|e)}{P(h_1|e)}$ iff $P(e|h_1) \leq P(e|h_2\&a)P(a|h_2)$

Theorem of Successful Consilience:

$$(5)\ \frac{P(h_1)}{P(h_2)} < \frac{P(h_1|e_1\&\ldots\&e_n)}{P(h_2\ \&\ a_1\&\ldots\&\ a_m|e_1\&\ldots\&\ e_n)}$$

iff

$$P\big(e_1\&\ldots\&e_n \,\big|\, h_2\&a_1\&\ldots\&\ a_n\big)P(a_1|a_2\&\ldots\&a_m\&h_2)\ldots P(a_n|h_2) < P(e_1\&\ldots\&\ e_n|h_1)$$

Proof: By Bayes' theorem:

$$\frac{P(h_1)}{P(h_2)} < \frac{P(h_1|e_1\&\ldots\&\ e_n)}{P(h_2\ \&\ a_1\&\ldots\&\ a_m|e_1\&\ldots\&\ e_n)}$$

Iff

$$\frac{P(h_1)}{P(h_2)} < \frac{\frac{P(e_1\&\ldots\&\ e_n|h_1)P(h_1)}{P(e_1\&\ldots\&\ e_n)}}{\frac{P(e_1\&\ldots\&\ e_n|h_2\&\ a_1\&\ldots\&\ a_m)P(h_2\&\ a_1\&\ldots\&\ a_m)}{P(e_1\&\ldots\&\ e_n)}}$$

By reasoning similar to the above proof:

$$\text{iff } \frac{P(h_1)}{P(h_2)} < \frac{P(e_1\&\ldots\&\ e_n|h_1)P(h_1)}{P(e_1\&\ldots\&\ e_n|h_2\&\ a_1\&\ldots\&\ a_m)P(h_2\&\ a_1\&\ldots\&\ a_m)}$$

Then, decomposing $P(h_2\&\ a_1\&\ldots\&\ a_m)$ into $P(a_1|a_2\&\ldots\&a_m\&h_2)\ldots P(a_m|h_2)P(h_2)$:

$$\text{iff } \frac{P(h_1)}{P(h_2)} < \frac{P(e_1\&\ldots\&\ e_n|h_1)P(h_1)}{P(e_1\&\ldots\&\ e_n|h_2\&\ a_1\&\ldots\&\ a_m)P(a_1|a_2\&\ldots\&a_m\&h_2)\ldots P(a_m|h_2)P(h_2)}$$

Which, by reasoning similar to the above, holds just in case:

$$P\left(e_1\& \ldots \& e_n \,\middle|\, h_2\&a_1\& \ldots \& a_n\right)P(a_1|a_2\& \ldots \&a_m\&h_2)\ldots P(a_n|h_2\,) < P(e_1\& \ldots \& e_n|h_1)$$

Therefore,

$$(5)\ \frac{P(h_1)}{P(h_2)} < \frac{P(h_1|e_1\& \ldots \& e_n)}{P(h_2\ \& a_1\& \ldots \& a_m|e_1\& \ldots \& e_n)}$$

iff

$$P\left(e_1\& \ldots \& e_n \,\middle|\, h_2\&a_1\& \ldots \& a_n\right)P(a_1|a_2\& \ldots \&a_m\&h_2)\ldots P(a_n|h_2\,) < P(e_1\& \ldots \& e_n|h_1)$$

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