



An Argument for the Principle of Indifference and Against the Wide Interval View

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Abstract

The principle of indifference has fallen from grace in contemporary philosophy, yet some papers have recently sought to vindicate its plausibility. This paper follows suit. In it, I articulate a version of the principle and provide what appears to be a novel argument in favour of it. The argument relies on a thought experiment where, intuitively, an agent's confidence in any particular outcome being true should decrease with the addition of outcomes to the relevant space of possible outcomes. Put simply: the greater the number of outcomes, the weaker your confidence should be in any one of those outcomes. The argument holds that this intuition favours the principle of indifference. I point out that, in contrast, the intuition is also incompatible with a major alternative to the principle which prescribes imprecise credences: the so-called *wide interval view*. Consequently, the argument may also be seen as an argument against the wide interval view.

Keywords Principle of indifference · Wide interval view · Objective Bayesianism

1 Introduction

We may be interested in credences for various reasons. Perhaps we want to understand or define rational decisions with reference to them, for example. In virtue of this or other interests, we may consequently be interested in principles that prescribe specific credences in various contexts. This paper investigates one of these principles for a particular context: the *principle of indifference*.

The principle concerns contexts involving *evidential symmetry*—that is, cases where our evidence bearing on the possible outcomes does not support one outcome more than any other (White 2010).

Here, we are particularly concerned with cases of evidential symmetry where one has no evidence about the so-called *objective chances* of the possible outcomes. To illustrate such cases, consider the following thought experiment from Kaplan (1996). Suppose you know that an urn contains balls, any one of which could only be either black or white. You know that a ball will be drawn from that urn. However, in Kaplan's words, you have

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“no evidence whatsoever about the proportion of black to white balls in the urn” (Kaplan 1996, 27, emphasis in the original). Like him, we might stipulate that you lack knowledge or evidence about the objective chances that the ball is of a particular colour. Now the concept of an ‘objective chance’ may seem mysterious, but the use of this term and others like it is well-established in philosophy.¹ Again following Kaplan, we could say that evidence about objective chances would include evidence about the proportion of black balls in the urn. We might also say that such evidence would include evidence about any tendencies of the mechanism that selected the ball to choose black balls. We can then suppose that you lack evidence about relevant chances in the urn case since you are not informed about such proportions or tendencies (or any other potentially chancy facts, for that matter). Let us call cases of this sort situations of *total stochastic ignorance*, situations where one lacks any evidence about relevant objective chances that bear on what one’s credences should be.² In a situation such as this, what should be your credence that the next drawn ball is black?

This paper provides an argument for one of two major contenders in the battle over how questions like this should be answered. I present the argument in Sect. 2 and I then defend it from various objections in Sect. 3.

But first, let us introduce the two contenders.

In one corner of the ring is the principle of indifference. The principle of indifference comes in many forms (Weisberg 2010; Strevens 2013; Meacham 2014). At least for now, we can understand the principle roughly as the norm that in evidentially symmetric situations of total stochastic ignorance, one should assign equal and numerically precise probabilities to the relevant outcomes. The principle perhaps offers the most initially intuitive response to my opening question about what one’s credence should be in the urn case. In the context of credences qua probabilities, the principle prescribes a credence of $\frac{1}{2}$ that the ball is black.³ But perhaps your credence should not be so precise.

In the other corner, then, is what has been called the *wide interval view*.⁴ Roughly speaking, the wide interval view is the norm that in evidentially symmetric cases of total stochastic ignorance, one’s credence should be maximally non-committal about the relevant outcomes. In a sense, then, one’s credal state should be spread over all of the possible probability values in the interval $[0, 1]$. Consequently, one’s credal state should be represented by not just one precise probability function, but rather by a *set* of probability functions that collectively assign to each relevant outcome every value in the interval $[0, 1]$. Such states are said to be *imprecise* in that they do not correspond to a precise credence.⁵ The set of functions which represent an imprecise credal state is called its *representor*. Applying this to our urn case, the wide interval view prescribes that your credal state be

¹ For example, White (2010) and Kaplan (1996) liberally use the terms “objective chance” or “objective probability” without any conceptual analysis of them, and perhaps rightly so. In Sect. 3.1, I will discuss issues involved in understanding objective chances and what it means to have evidence about them.

² I have chosen the term “stochastic ignorance” for the concept since already established nomenclature for it is lacking and the concept denotes ignorance of the kind of probabilities that concern what Howson and Urbach call a type of “stochastic (random or chance-like) experiment” (Howson and Urbach 2006, 15, emphasis in the original).

³ Given that the subjective interpretation of probabilities as credences dominates formal epistemology, I sometimes use the terms “probability” and “credence” interchangeably in this paper.

⁴ See this terminology in, for instance, Rinard (2013).

⁵ For concepts that are more or less similar to that of imprecise credences, see Levi (1974), van Fraassen (1990), Walley (1991), Joyce (2005), Sturgeon (2008), White (2010), and Elga (2010).

maximally non-committal: it is representable with a representor that contains the set of all probability functions over the outcomes where the next drawn ball is either black or white.

Some philosophers have endorsed the wide interval view, with Kaplan (1996) perhaps being the most explicit example.⁶ In the urn case, he asks you to consider what credence you should have in the hypothesis that the next drawn ball is black in comparison to the hypothesis that it will be white. Where $con(-)$ is a function representing a degree of confidence, he states:

If you have no reason in [this case] to assign one of the two hypotheses a higher value of $con(-)$ than the other, it is only because you have no reason to assign either of the hypotheses any particular value at all. But then it would seem that the appropriate response to [this case] would be to acknowledge as much and refrain from assigning a value to either hypothesis. (Kaplan 1996, 27)

This clearly contradicts the counsel offered by the principle of indifference. Kaplan then affirms his view that “when you have no evidence whatsoever pertaining to the truth or falsehood of a hypothesis P , then, for every real number n , $1 \geq n \geq 0$, your set of con -assignments should contain at least one assignment on which $con(P) = n$ ” (Kaplan 1996, 28). This would mean, then, that one’s credal state is best represented with a representor of functions collectively assigning every value in the interval $[0, 1]$ to the hypothesis that the next drawn ball is black.

Aside from proponents of the wide interval view, there are others such as myself and White who consider it to be a *prima facie* plausible answer to the question of what to do in situations of total stochastic ignorance.

So the wide interval view and the principle of indifference are two candidate answers about what to do when confronted with stochastic ignorance, but another alternative is sharp subjectivism: the view that one can have any single and precise probability as their credence so long as it is permitted by the evidence—albeit perhaps with the exception of extreme values. However, I will not discuss subjectivism here, both for lack of space and because of my opinion that some of White’s (2010) arguments are sufficiently compelling so as to obviate a discussion of the subjectivist alternative to the principle of indifference. Hence, I will focus on the principle of indifference and what I think is its most plausible competitor—the wide interval view.

There are many considerations for and against the two contenders. Philosophers have marshalled arguments against both the principle of indifference (Kyburg 1974; Levi 1985; Kaplan 1996; Joyce 2010) and the wide interval view (Elga 2010; White 2010). Some have also presented arguments for the principle of indifference, including Pettigrew (2016), Williamson (2010), Jaynes (1957), Paris and Vencovská (1990) and Hawthorne et al. (2017). Clearly, the considerations bearing on the debate are so numerous that a paper of this size cannot thoroughly survey them all. Hence, I will focus on some to the exclusion of others.

In this respect, four arguments in the literature have special relevance to this paper. White’s (2010) so-called “argument from cases” and his “evidentialist argument” bear some similarities to this paper’s main argument since these arguments assert the intuitive rationality of symmetrical credences given symmetrically balanced evidence, especially in the light of particular thought experiments involving prizes behind doors, detectives and

⁶ Rinard claims that Walley (1991), Joyce (2005), and Weatherston (2007) are also of the opinion that, roughly speaking, “in a case of no evidence, rationality requires credence to be spread over $[0, 1]$ ” (Rinard 2013, 157). However, a thorough discussion of their opinions is beyond the scope of this paper.

the like. Yet neither of White's arguments attempt to discredit the wide interval view as an alternative to the principle of indifference. For this reason, White appeals to his coin puzzle to disfavour the wide interval view, a puzzle that has generated much controversy (see Topey (2012), Dodd (2013), and Pedersen and Wheeler (2014)). Unlike these two arguments, this paper appeals to variations in the size of the space of possible outcomes to favour the principle of indifference over the wide interval view.⁷ Rinard's (2013) argument is also similar to this paper's argument since it focuses on comparative levels of confidence in order to undermine the wide interval view. Despite this, however, Rinard's argument does not support—or even mention—the principle of indifference, unlike the argument of this paper; so it is essentially different in its intent. A fourth argument of special relevance is White's (2010) "argument from statistical inference", but I will postpone the discussion of this argument until Sect. 3.9 where its relevance will, I think, become readily apparent.

The task of this paper, then, is to offer what looks to me like a novel argument to favour the principle of indifference over the wide interval view.⁸

2 The Argument

2.1 A Claim and a Disclaimer

Before presenting this paper's main argument, I should first clarify what I am and am not arguing for.

⁷ One might think of the "space of possible outcomes" in terms of a standard sample space which contains *all* possible combinations of outcomes that are a priori constructible in a given first-order language (including those specifying that the ball is black *and* the ball is white). While the argument could be made with this notion of a space of possible outcomes, this is not the way I am thinking of a space of possible outcomes. Instead, I use the term to refer to the set of outcomes which are consistent with what the agent knows, and later on, I shall talk of two spaces of possible outcomes: one for the colour of one ball and one for the colour of another.

⁸ I believe this argument is probably novel—or at the very least, not widely-known—for several reasons. I could not find an argument of this sort when searching the literature. This might be explained by the fact that the imprecise credence view which it bears on is somewhat of a newcomer to philosophy, perhaps only emerging in the last 25 years. Furthermore, twenty-one academics have given feedback on the argument (all of whom either were reviewers of earlier versions of this paper or were formal epistemologists I spoke with in person), and most of them did not object to its novelty. However, seven academics did have concerns about its originality. Some thought the argument was similar to White (2010) and Rinard's (2013) arguments. But, as I have argued in various places in this paper, there are specific reasons why their arguments are substantially different to that of this paper. Others instead had qualms about the originality of the argument, not necessarily because it had been discussed in the literature, but because its key points may seem quite simple or obvious to people regardless. For example, one thought that the argument's key intuition is an immediate consequence of the principle and consequently may have been inexplicitly understood by others as a consideration in favour of the principle. Regardless, it still looks like the literature lacks an explicit discussion of the extent to which this intuition supports the principle of indifference over the imprecise view. This discussion is not found, for example, in White's (2010) review of arguments for the principle of indifference, nor in his argument against the wide-interval view in that same paper. Yet it seems to me that such a discussion is warranted, at the very least because some accept the value and force of the intuition while others object to it—that is, if the feedback I have received is anything to go by. Consequently, I have tried to rectify this situation by explicitly defending the force of the intuition and by addressing various objections later in the paper. In any case, it may be that the main argument of this paper is not obvious at first, although it may strike one as such after it has been pointed out; after all, it was not obvious to me at first, even after reading the relevant literature. Regardless, it is possible that the literature already contains an argument of this sort, but that is the case with virtually any new argument. So I am inclined to treat the novelty of the argument as innocent until proven guilty.

I am not arguing for what we might call a *general principle of indifference*, a principle asserting that for every evidentially symmetric case of total stochastic ignorance, one should assign equal and precise probabilities to the relevant outcomes. Such a principle faces problems since there are allegedly cases where contradictions arise if one assigns equal and precise probabilities to the relevant outcomes according to different partitions of the outcomes. For example, suppose I tell you that I have a marble in my hand, and you are totally stochastically ignorant about its colour. What is the probability that it is green? We can partition the space of possible outcomes as per the set {the marble is green, the marble is not green}. If each outcome in this partition is assigned the same precise probability à la principle of indifference, then the probability that the marble is green is $\frac{1}{2}$. But another way to partition the relevant outcomes is the set {the marble is green, the marble is red, the marble is neither green nor red}. If each outcome in this partition is assigned the same precise probability, then the probability that the marble is green is $\frac{1}{3}$ instead of $\frac{1}{2}$. And there are many other possible partitions, each potentially leading to conflicting probabilistic judgments. So there are cases where there arguably is no partition of the outcomes that is in some sense uniquely correct. (I will further discuss the notion of a uniquely correct partition in Sect. 3.8.)

Consequently, the principle I argue for is restricted insofar as it concerns *only uniquely correct partitions* in spaces of finitely many outcomes:

Restricted principle of indifference: In evidentially symmetric cases of total stochastic ignorance where there is a uniquely correct partition of finitely many possible outcomes, assign equal and precise probabilities to each possible outcome.

From here on, I will refer to this restricted principle of indifference simply as “the principle of indifference”.

2.2 The Main Argument

The argument is relatively simple, and it bears some similarities with Rinard’s (2013) argument, as mentioned earlier. Here is a thought experiment to illustrate the argument. Suppose I prepare two urns of balls, urn_1 and urn_2 , and then I place them in front of you. I tell you that every ball in urn_1 may be either black or white, and so it contains no balls of other colours. Furthermore, I tell you that any given ball in urn_2 could be black, white, orange, blue, red, yellow, magenta, green, brown or grey. Urn_2 could contain no other colours. I tell you nothing of the ratio of the coloured balls in the urn. (And we can suppose that you have good grounds to trust that my testimony is truthful.) It is consistent with what I have said that all of the balls in urn_2 are orange, since all of them might have been any of the ten aforementioned colours, but it turns out that all of them are orange. Likewise, all of the balls in urn_2 might be white whereas all of the balls in urn_1 might be black. Suppose you also have no evidence about such things as any tendency of mine to place balls of certain colours in urns.

In this case, we could say I am providing you with information that enables you to rule out some epistemic possibilities while retaining others. In this sense, a proposition p is epistemically possible for you just in case, for all you know, you cannot rule out p . For example, it is epistemically possible that all of the balls in urn_2 might be magenta. This is because, for all you know, they could all be magenta, and this would still be an epistemic possibility even if it happens that all of the balls from urn_2 are orange (provided that you

do not know this). In contrast, it is not epistemically possible that all of the balls in urn_1 are magenta, since you have received information from me to rule this out. This is the sense of possibility that I have in mind when I say that something is possible.

Now suppose I draw one ball from each urn and we call the balls $ball_1$ and $ball_2$ for urn_1 and urn_2 respectively. As you know, $ball_1$ could be black, but it could also be white. $Ball_2$ could be black too, but it could also be white, orange, blue, red, yellow, magenta, green, brown or grey.

Here is the important question then: should you be less confident that $ball_2$ is, say, black than that $ball_1$ is black? Intuitively, the answer is affirmative: you should be less confident that $ball_2$ is black than that $ball_1$ is black. After all, there are only two possible outcomes for what colour $ball_1$ may be: black or white. Yet there are ten possible outcomes for what colour $ball_2$ may be: orange, blue, red, yellow, magenta, green, black, white, brown or grey. The proposition that $ball_2$ could be, say, magenta has to be countenanced no less seriously than the possibility that it is black or any of the other eight colours. For each ball, you have no reason to favour one outcome over the other, yet the additional possible colours that $ball_2$ could be arguably suggests that you should be less confident that $ball_2$ is black than that $ball_1$ is black.

This illustrates the general and key intuition of the argument: you should be less confident that $ball_2$ is any particular colour than that $ball_1$ is any particular colour by virtue of the additional outcomes for what colour $ball_2$ might be. Note that this does not apply only to considerations of blackness: you should also be less confident that $ball_2$ is, say, magenta than that $ball_1$ is white. (And if this intuition has not emerged for you, then we could see whether it emerges when the thought experiment is modified so that urn_2 could contain, say, a hundred distinct colours instead of ten.)

To further drive home the intuition, suppose I tell you that you can pick a ball and that if it is black, then you win a million dollars. Now which ball should you pick? Presumably the answer is that you should pick $ball_1$. This is best explained in part by the key intuition that you should be less confident that $ball_2$ is any particular colour—such as black—than that $ball_1$ is any particular colour by virtue of the additional outcomes for what colour $ball_2$ may be.⁹

Although I have spoken with many who have this intuition, some may not have it, in which case this argument would have little value for them. Regardless, it may have some value for those who do have the intuition, including myself and certain commentators on the argument.

Assuming that you do have the key intuition, though, where does its normative force come from?

One explanation is that you have what we might call tacit *stochastic evidence* or *knowledge* which, in this context, is evidence or knowledge about any objective chances that the balls are black. Yet this explanation is untenable according to the stipulations of the thought experiment. It is not as if, for example, you know the ratio of colours of balls in the urns or know of some tendency I have to place balls of certain colours in urns. For all you know, I could have a habit of or preference for putting balls of a few colours in urns, or perhaps the opposite is true. So it appears that you have a situation of total stochastic

⁹ Of course, this is not a pragmatic argument for the principle of indifference. It is an argument for the principle as an epistemically rational constraint on credences. But the point is that pragmatic considerations (about which choice to make) reflect deeper epistemic intuitions about how confident we should be that one ball is black compared to the other.

ignorance, in which case something aside from stochastic evidence or knowledge must explain the normative force of the intuition.

So does the wide interval view explain the normative force of the key intuition? Arguably, no. The wide interval view would recommend that you be maximally and equally non-committal about the outcomes for both balls, thus having a credal state that is representable with a set of functions collectively assigning each outcome every value in the interval $[0, 1]$. The reason for this is that, as Kaplan (1996) would say, you know nothing of the ratio of coloured balls in the urn and you supposedly have no reason to think that any ball is more likely to be one particular colour rather than another. According to the view, then, you should not be less confident that, say, *ball*₂ is black than that *ball*₁ is black. In this sense, the intuition is incompatible with the wide interval view.

I suggest, then, that the intuition has normative force, and that this normative force supports the principle of indifference for three reasons.

The first of these is that the principle of indifference is arguably the end product of *distilling* our intuitions about the thought experiment from irrelevant details. We could re-run the thought experiment and vary the details each time, such as details concerning the nature of the objects. We could, for instance, change all of the balls into cubes and permute the colours of the balls to different colours for the cubes, all while preserving the overall number of colours that the respective cubes might be. For example, *urn*₁ might instead contain cubes which could be either red or blue while *urn*₂ might also contain cubes which could be any of ten different colours. This change in details would preserve a more general intuition underlying both the original thought experiment and its modification: that you should be less confident about the outcome regarding the colour of the object drawn from *urn*₂ than about the outcome regarding the colour of the object from *urn*₁. And we could vary many other details of the thought experiment if we had enough time—all while preserving the intuition at some level of abstraction.

Varying these details would not matter for what our credences should be *unless* these are about the *important* details which *do* matter. Some of these important details are about whether you are totally stochastically ignorant and in a case of evidential symmetry. Suppose, for example, that we changed the details of the thought experiment so that instead of it concerning two urns—one potentially containing two colours and the other potentially containing ten colours—the experiment instead concerned two towns, with *town*₁ containing two restaurants and with *town*₂ containing ten restaurants. Suppose you know your friend Bob had lunch in the two towns on two separate occasions, and you are wondering which restaurant he ate at in each town, but you know that he has historically been inclined to eat at a particular restaurant franchise in *town*₂, unlike any particular restaurant in *town*₁. Here, then, you would have stochastic evidence which should affect what your credences are, thus overriding the initial intuition and thus suggesting that stochastic ignorance and evidential symmetry are some of the details of the original thought experiment that matter.

If we were to change other details about the thought experiment, I think we would also find that two other kinds of details matter as well. One of these is about the *number of outcomes*; for the greater the number of evidentially symmetric outcomes, the smaller your confidence should be in any particular one of them.¹⁰ The other kind of details that matter is about whether there is a *uniquely correct partition of outcomes*, since some partitions

¹⁰ This, however, is if the number of outcomes is finite. Since the restricted principle of indifference only concerns finitely many outcomes, I have nothing to say about infinite sets of outcomes.

of outcomes—like those in Bertrand-style paradoxes—are not obvious candidates for equal and sharp probabilities (and I will say more about such partitions in the Sect. 3.8.).

My point is that we can take intuitions about the original thought experiment involving the urns, and then we can distil these intuitions by sifting out the features of them that matter—such as stochastic ignorance—from the features that do not—such as the shape of the objects. In doing this, I think we will find that the features that matter are the presence of evidential symmetry, stochastic ignorance, and a uniquely correct partition, as well as the relevant number of outcomes. These features then have implications for what attitudes we should take towards the outcomes of a uniquely correct partition. And intuitively such attitudes should satisfy a symmetry constraint: intuitively, we should take the same attitude toward each outcome since we have the same evidence (or lack thereof) about each outcome. Yet when we take our intuitions about the thought experiments—each with their normative force—and remove the irrelevant details, we find that they boil down to a central intuition which itself carries normative force and which encapsulates only the important details. This intuition, I believe, is the principle of indifference:

In evidentially symmetric cases of total stochastic ignorance where there is a uniquely correct partition of finitely many possible outcomes, assign equal and precise probabilities to each possible outcome.

So the first reason involves adjusting *hypothetical* details of thought experiments to thereby distil the epistemically relevant features of our intuitions, something that could in principle be done without ever needing to actually assign a probability to any real-world event.

Having distinguished the relevant from the irrelevant details, a second and related reason to adopt the principle of indifference arises from the requirement to be *coherent* across our *actual* probability assignments. Of course, this would not be coherence qua having probability assignments which conform to the probability calculus. Instead, it would be what we might call *rationale* coherence, assigning probabilities on the basis of rationales which cohere with each other. For example, suppose Jeff has sharp and symmetrical probabilities on a Tuesday when he is asked about his confidence about the balls from the urns. However, suppose that, the next day, Jeff is maximally non-committal about his respective probabilities for a similar situation about cubes, even though (as he believes) it is the same as Tuesday's case in all the important respects—namely, by involving an evidentially symmetric case of stochastic ignorance, a uniquely correct partition and the same number of outcomes. Here, it looks like Jeff is guilty of a kind of rationale incoherence: he has different probability assignments for the balls and for the cubes, and he presumably would have rationales for both assignments which fail to cohere if there are no relevant differences between the situations which would call for different rationales. Instead, since the cases are the same in all the important respects, they should have the same probability assignments underpinned by the same rationales—and the principle of indifference would provide the same rationales in both cases. If Jeff were to endorse the principle of indifference and to act in accordance with it, then this would help him to avoid such incoherence. For this reason, then, the requirement to be coherent impels us to accept the normative force of principles which are more general than any intuitions about specific inferences which accord with those principles, intuitions such as those about the balls in the urn. This then is one reason to accept the principle of indifference rather than merely accepting the key intuition of the thought experiment.

Aside from these considerations about distillation and the requirement of coherence, a third reason to accept the principle of indifference is that it can *explain* well the normative

force of the intuitions of the thought experiment. We often regard some evidence as providing support for a theory to some extent if that theory can explain the evidence well, and this is largely a matter of the theory leading us to expect the evidence. For example, various genetic similarities across species provided support for evolutionary theory since evolutionary theory could explain such similarities well by leading us to expect them. Similarly, the anomalous precession of Mercury provided support for Einstein's theory of relativity since relativity theory could explain and predict the anomalous precession of Mercury. And likewise, we might think that the principle of indifference receives support from the intuitions of the thought experiment because the principle explains why the intuitions of the thought experiment have normative force: if the principle of indifference was a correct principle with normative force, we would expect such intuitions about the thought experiment to have such force too.

In summary, then, the argument is as follows. The thought experiment gives rise to the key intuition that you should be less confident that *ball*₂ is any particular colour—such as black—than that *ball*₁ is any particular colour by virtue of the additional outcomes for what colour *ball*₂ may be. By stipulation, stochastic evidence does not explain this intuition, and the intuition is also incompatible with the wide interval view. However, the key intuition (and related intuitions) supports the restricted principle of indifference for three reasons. One of these is that the principle of indifference is the end product of distilling our intuitions about the thought experiment from irrelevant details, taking into account only the features that matter for what our credences should be. Relatedly, the second reason is that the requirement to be coherent across our probability assignments favours accepting the principle of indifference as a general guide to inferences about the cases which it concerns. A third reason is that, arguably, the principle of indifference can explain well the intuitions about the thought experiment.

3 Objections

When presenting this paper's main argument in the past, I have received sympathy from some commentators, but I have also encountered a staggering variety of objections to it from others—many of which criticize the argument in quite different and sometimes incompatible ways. While I cannot survey all such objections here for lack of space, I will instead focus on the ones which have been repeatedly raised or which appear salient to me.¹¹ Hence, I will consider two sets of objections. The first set objects to some premises of the argument, namely, those about the intuitions of the main thought experiment or about the ability of such intuitions to be accommodated by stochastic evidence, the wide interval view or the principle of indifference. The second set objects to the significance of the argument.

¹¹ To give one example of another objection I have not discussed here, one critic claimed that we could try to save the wide interval view by adopting an "overall confidence level" which is a function of the functions in the representor and which somehow assigns a low degree of confidence to the outcomes. I have a response to this objection and others in a longer version of this paper. There, my response to this objection is that having a sharp and low overall confidence level is inconsistent with the motivation for the wide interval view—namely, to have no particularly committal confidence level at all.

3.1 Tacit Stochastic Evidence or Knowledge

One objection is that although the key intuition holds and you should be less confident that, say, $ball_2$ is black, this is only because you are not truly totally stochastically ignorant. Instead, you have some tacit stochastic evidence or knowledge, and so the principle of indifference does not even apply in this case. Although I briefly argued that this was not the case when presenting this paper's main argument, let us more carefully consider the subject here.

To evaluate this objection, one would perhaps ideally have a conceptual analysis of what objective chances are in order to specify what it means to have knowledge or evidence about them. However, providing such an analysis is problematic and cannot be done rigorously in a brief paper focusing primarily on an argument for the principle of indifference (see some of the relevant complexities discussed in Hájek (2012)). Consequently, I follow Kaplan (1996), White (2010) and others in discussing objective chances without defending a conceptual analysis of them.

Nevertheless, there is a case to be made that one is stochastically ignorant in the urn thought experiment, at least according to the presumably uncontroversial candidates for what it would mean to have stochastic evidence or knowledge. For example, we might say, following Kaplan (1996), that evidence about objective chances could include evidence about the ratio of coloured balls in the urns. Another candidate for stochastic evidence may be frequency data that indicates that, for example, people like me tend to present urns with balls of a particular distribution of colours. Other potential candidates for stochastic evidence may resemble paradigmatic cases of stochastic evidence, such as evidence about the chance of a tossed coin landing on heads or of some quantum event.

In any case, we can suppose realistically that you lack any of these candidates for stochastic evidence: evidence about ratios of coloured balls, my tendency to present urns containing balls with a particular colour distribution or evidence somehow resembling paradigmatic evidence about coin tosses, quantum events, and the like.

Since we are doing a thought experiment, though, I can also ask you to think of any of your preferred alternative candidates for what it means to have stochastic evidence or knowledge (if you have them) and to then suppose that, in the experiment, you lack any such evidence or knowledge according to those candidates. We can thus suppose it out of existence, as it were.

In any case, supposing that you lack stochastic evidence or knowledge according to whatever conception one has, I suspect that the key intuition would hold: despite you lacking this evidence or knowledge, you should be less confident that $ball_2$ is any particular colour than that $ball_1$ is any particular colour by virtue of the additional outcomes for what colour $ball_2$ may be. In this case, then, the objection that the intuition relies on tacit stochastic knowledge or evidence is untenable.

Of course, one response to the argument might be to redefine objective chances—or evidence about such chances—so as to include other things that would make you no longer stochastically ignorant, thereby entailing that the principle does not apply. I am not going to survey all such possible definitions, particularly when any given definition may be controversial and not actually held by philosophers in the relevant debates, such as Kaplan. But I will discuss one example: suppose we define evidence of objective chances so as to include knowledge of the number of possible outcomes in a partition. Of course, if we defined it as such, then we would have knowledge of objective chances in the urn thought

experiment. Thus, the principle of indifference would not apply to this case *if* we adopted this redefinition, since the case would fail to qualify as one involving stochastic ignorance.

My response to this move would not be to object to this redefinition, but rather to point out that the spirit of this paper's argument would still survive despite it. The purpose of the argument is to show that the sharp and symmetrical credences are warranted in a particular kind of situation where others like Kaplan clearly think they are not. This kind of situation can be specified independently of how we understand the notion of objective chances: it is a situation where one lacks evidence about such things as the ratio of balls in the urns for instance. Some people, like Kaplan, think these situations *lack* evidence about objective chances because they endorse a particular conception of what such objective chances are, such as the ratios of coloured balls in this case; but others might think these situations *do* involve evidence of objective chances if they endorse the above redefinition of objective chances. Although it is worth exploring which conception—if any—is the right conception, that is not the point of this paper. The primary point of this paper is to address the contentious question about whether sharp and symmetrical credences are warranted in these situations, and the principle of indifference is typically characterised as the view that such credences are warranted as such. However, the primary point of the paper is not one about whether we should label such cases as involving “ignorance of objective chances” given one conception or another. Regardless, I have used such a label since Kaplan and others think such situations do lack evidence about objective chances (according to their conception), and I have so far seen no one in the literature with an opposing view.

So the point is that the argument will achieve its ultimate purpose if it shows that sharp and symmetrical credences are warranted in the situations where Kaplan and others think they are not—and this is regardless of how we define objective chances, stochastic ignorance and related notions.

3.2 Unknown Facts

Another objection is that the principle of indifference should not be accepted on the basis of the intuition since there might be facts which would mandate probabilities that conflict with the principle's prescriptions if you were to know of those facts. Perhaps, for example, unknown to you, urn_2 contains only black balls and urn_1 contains only white balls, or perhaps I have some kind of tendency to favour urns with balls that are mostly black. These are facts which presumably should change your credences for the propositions about the colour of the balls if you were to know them. The objection, then, is that while you do not know of such facts, assigning equal and precise probabilities amounts to assuming that there are no such unknown facts, but clearly such an assumption is unwarranted.¹²

However, we are in the business of probability, a business where one often assigns probabilities despite the possibility that some facts which are unknown to the assigner would change the relevant probabilities were she to know of them. For example, suppose Jenny knows that the empirical relative frequency or proportion of Mexicans with brown eyes is 99%, and she learns that a particular Mexican is visiting her eye clinic. If she has no other relevant information, then she arguably can be highly confident that the Mexican's eyes are brown. This is despite the possibility that there are facts, unknown to Jenny, which should change her credence that the Mexican has brown eyes if she were to know of them.

¹² See a similar response discussed in White (2010).

Perhaps, unknown to Jenny, the Mexican belongs to the Jenkins family, all of whose members have blue eyes. This is a fact which, if known, would instead mandate a credence of 0 that the Mexican has brown eyes.¹³ The point is that probabilistic reasoning generally allows us to assign probabilities despite the possibility that some facts would change the relevant probabilities were we to know of them; hence, we should not find it objectionable that the principle of indifference allows this too.

The critic might object that there is some disanalogy between probabilities based on the principle of indifference and probabilities based on empirical relative frequencies (as in the case of Jenny). But I cannot see a promising disanalogy to be given.

For instance, the critic might object that the principle of indifference somehow conjures up knowledge of probabilities from ignorance and they might insist that empirical frequency-based probabilities avoid such conjuration.

But arguably an analogous objection could be taken up against certain probabilities based on empirical frequencies. For instance, one might argue that such probabilities involve conjuring up knowledge to the effect that, say, the Mexican is a randomly-selected member of the population of Mexicans, thereby warranting a probability of 99% that the Mexican has brown eyes.

In fact, Levi (1977) did raise a similar objection, yet it failed to convince Pollock (1990), Kyburg and Teng (2001), White (2010), Thorn (2012), and certain others who have written about frequency-based probabilities. (For explicit responses to this objection, see Pollock (1990), Thorn (2012), and White (2010).) The reason for this is that it is both a widespread and intuitively rational practice to assign empirical frequency-based probabilities, and this is despite the possibility of there being facts concerning such things as selection bias which would change the probabilities were one to know of them.

Similarly, I think that the assignment of equal and precise probabilities when one is stochastically ignorant is also a fairly widespread and intuitively rational practice, and this is despite the possibility of there being facts which would change the probabilities were one to know of them.

3.3 Distillation, Explanation, and Other Principles

I claimed that the key intuition of the thought experiment (and others) supports the principle of indifference for three reasons. One of these reasons is that the principle of indifference is the end product of distilling our intuitions about the thought experiment from irrelevant details, taking into account only the features that matter when it comes to what our credences should be. Another reason is that, arguably, the principle of indifference can explain well the key intuition (and others) about the thought experiment.

One might object to this claim—as one commentator has—stating that there are alternative principles which could be arrived at via the process of distillation or which could explain the key intuition. For instance, the said commentator claims to offer an example of such a principle, the principle that “black is strictly less probable for the ball from urn 2 than it is from urn 1”. They further note that this principle does “not amount to the principle of indifference at all”.

¹³ This would be a probability assignment in conformity with the principle of the narrowest reference class. For more on the preference for narrower or more specific reference classes, see Thorn (2017).

This objection, however, does not undermine the claim that the intuitions of the experiment support the principle of indifference. The principle of indifference can have normative force because of considerations such as distillation and explanation, but so too can the putative principle that “black is strictly less probable for the ball from urn 2 than it is from urn 1”. What matters is not that other principles lack normative force per se, but rather that the would-be normative force of these other principles somehow *detracts* from the normative force of the principle of indifference. An alternative principle could detract as such if it made prescriptions which conflict with the principle of indifference, if it explained all of the intuitions about the thought experiment just as well and if it was plausibly the end product of distillation—that is, if it satisfies the symmetry requirement and encapsulates all and only those details that matter in the thought experiment and its modifications. To recap, I have claimed that such details are details about evidential symmetry, stochastic ignorance, the presence of a uniquely correct partition and the number of outcomes—and not details about, say, the colour of the balls or the shape of the objects.

Of course, if the critic thinks there is a genuinely defensible alternative principle which explains the intuitions and which survives distillation just as well as the principle of indifference, then I would be eager to see it.¹⁴

Regardless, I can see no such alternative principle which is defended in the literature, and this paper’s main argument then brings a pertinent consideration to bear on some live options for rational constraints in the face of stochastic ignorance.

In saying that, another commentator has proposed an alternative theory which purportedly can accommodate the key intuitions of the thought experiment, and I will turn to consider this in the next subsection.

3.4 The Open Interval View

The critic might object to the argument by claiming that the wide interval view can accommodate the argument’s key intuition with a simple modification, one which transforms it into what we can call the *open interval view*. This is the view that in cases of total stochastic ignorance, one’s credence should be largely non-committal about the relevant outcomes and should in some sense be spread over all of the possible probability values in the *open interval* (0, 1). Put differently, the open interval view is just the wide interval view, but with the omission of 0 and 1 as permissible probability assignments to represent one’s credence. The critic might think such a view can accommodate the intuition that you should be less confident that *ball*₂ is black than that *ball*₁ is black. In particular, since 0 is not a permissible probability assignment, every probability assignment will be greater than 0, including the probability assigned to the proposition that *ball*₁ is black. In this case, every probability

¹⁴ Of course, one might think that the principal principle (Lewis 1980), or something like it, is one such alternative principle—at least if one defines objective chances so as to include the number of outcomes in a uniquely correct partition when no other evidence favours one outcome over another. Given this definition, the principle would prescribe that we match our credences to the chances qua the proportions of outcomes where a given proposition is true. I have not seen anyone in the literature who would adopt such a definition and consequently think that the principal principle applies as such, but even if they did so, I argued in Sect. 3.1 that this paper’s argument would still achieve its main purpose: showing that sharp and symmetrical credences are warranted in cases where others (notably Kaplan) think they are not. But granted, there are relevant background questions about how to define objective chances and the principle of indifference, although, as mentioned in Sect. 3.1, these are not the focus of this paper.

function in one's representor can assign a value to the proposition that $ball_2$ is black which is lower than the value it assigns to the proposition that $ball_1$ is black.

I think that this response is problematic for several reasons, but I will only dwell on one of these here.¹⁵

Although the open interval view is consistent with the notion that you should be less confident that $ball_2$ is black than that $ball_1$ is black, it is inconsistent with the general intuition which this notion illustrates: you should be less confident that $ball_2$ is *any particular* colour of its ten possible colours in comparison to the two possible colours for $ball_1$. For example, on the open interval view, there will be some function assigning a probability of, say, 0.1 to the proposition that $ball_1$ is black, since the view claims that for every number in the interval (0, 1), there should be some function that assigns that number to that proposition. According to the open interval view, by the critic's hypothesis, we assume that you are less confident that $ball_2$ is black than that $ball_1$ is black, and this means that this same function will assign a probability to $ball_2$ being black which is lower than 0.1. However, by the probability calculus, you will be more confident that $ball_2$ is some particular color of the other nine possible colours than you are that $ball_1$ is black (proof is in the footnote).¹⁶ This illustrates that the open interval view cannot accommodate the argument's key intuition: that you should be less confident that $ball_2$ is any particular colour than that $ball_1$ is any particular colour.

3.5 Untrustworthy Intuitions

Another objection to the argument is that it relies on untrustworthy intuition. More specifically, it relies on various intuitions, especially the intuition that you should be less confident that $ball_2$ is any particular colour than that $ball_1$ is any particular colour by virtue of the additional outcomes for what colour $ball_2$ might be. Yet we know all too well that intuitions can occasionally be unreliable, as is illustrated by the heuristics and biases research programme.¹⁷ Why, then, should we trust the intuitions that are appealed to in this paper's main argument?

In response, note that an objection of this sort could be levelled against many theories of probabilistic and statistical inference, since these theories are often—if not always—supported by intuitions. For example, Bacchus et al. articulate a theory by which one can determine probabilities on the basis of statistical information, and they argue for this theory by showing how it accords with various intuitions about reasonable inferences (Bacchus et al. 1996, 79). An example of such an intuition is the intuition about *basic direct inference*. This is the intuition that, to take Bacchus et al.'s example, if you know that Eric has jaundice and that 80% of people with jaundice also have hepatitis, and if you have no other knowledge about the probability that Eric has hepatitis, then your credence that Eric

¹⁵ More specifically, other problems arise from the possibility that the open interval response is ad hoc, less simple and invokes a representor of functions which could not adequately represent any *actual* credal state since it's unclear as to what it would mean to have a credal state of confidence that conforms to the view (unlike precise probabilities and the representor of the wide interval view).

¹⁶ In particular, if a probability less than 0.1 is assigned to the proposition that $ball_2$ is some particular colour, then a probability higher than 0.1 must be assigned to $ball_2$ being some *other particular* colour. Otherwise, the sum of the probabilities for the ten colours would be less than 1, thereby violating the probability calculus. Then, this assignment will be higher than the probability that $ball_1$ is black, thus contradicting the general intuition of the thought experiment: you should be less confident that $ball_2$ is *any particular* colour of its ten possible colours in comparison to the two possible colours for $ball_1$.

¹⁷ For a concise and insightful discussion of the programme, see O'Hagan et al. (2006, 31–52).

has hepatitis should be 0.8. Aside from Bacchus et al., Jaynes (1976) appeals to intuition as well, since they rail against classical methods of statistical inference by claiming that they contradict the intuition enshrined in common sense regarding rational statistical inferences. He thus states, “[i]t has been recognized by all, beginning with Laplace, that the purpose of a statistical analysis is to aid our common sense by giving a quantitative measure to what we feel intuitively” (Jaynes 1976, 218). Jaynes and Bacchus et al. work with a Bayesian notion of probability as credence, but frequentists also appeal to intuition in support of their theories. For example, Kyburg and Teng defend the semantics of their theory of evidential probability, and they state that “the true test of our semantics requires seeing whether it yields intuitively plausible results in simple cases, to begin with, and reasonable results in more complex cases in which our intuitions are not so strong” (Kyburg and Teng 2001, 244). Here, an example of an intuition about a simple case would include, for instance, the sort of intuition about basic direct inference that Bacchus et al. appeal to, as mentioned. Such appeals to intuition also jibe with Goodman’s (1983, 64) view that the rules of logic are likewise acceptable because they accord with what are arguably intuitions about particular instances of valid inferences.

The takeaway message is that reliance on intuition is also a problem for other theories of probabilistic inference, and, to echo a theme from Hájek (2003), there is a sense in which everybody’s problem is nobody’s particular problem at all. Like these other theories, this paper’s argument relies on particular intuitions about rational inferences, and, like these other theories, the question could arise as to what would justify trusting *these particular intuitions*. Like these other theories, however, I have no answer to give, for I think that an appeal to foundational intuitions will at some point be inevitable for any theory of inductive inference.¹⁸ But like these other theories, I suspect many will share the core intuitions of the argument regardless, and that this will be good enough for the argument to have some value for them. Of course, some intuitions are untrustworthy, just as some formal methods of statistical inference are arguably untrustworthy (take Ziliak and McCloskey’s (2008) forceful critique of null-hypothesis significance tests, for example). But this fact by itself does not justify full-blown distrust of intuition in general any more than the untrustworthiness of some formal methods justifies distrust of all formal methods.

Unless the critic can give some reason to distrust the argument’s specific intuitions, then, I think it is appropriate to regard such intuitions as defeasible sources of support for the principle of indifference—just as philosophers and statisticians do for many other theories of probabilistic inference. (Of course, one might think that the multiple partitions problem or related concerns constitute such a reason, but I will address these concerns in Sect. 3.8.)

3.6 Begging the Question

One might object that the key premise and intuition of the paper is circular—the premise begs the question because it is unacceptable to someone who did not already believe the conclusion which the premise supposedly supports. (And just to refresh our memories, the

¹⁸ In saying that, I suspect that the restricted principle of indifference is more gradationally accurate than any competing candidate principles, and that there may be evolutionary reasons why humans have intuitions which accord with the principle’s prescriptions. However, I do not have strong evidence to support this suspicion which could also justify trusting the intuitions of the thought experiment.

key premise is that that you should be less confident that *ball*₂ is *any particular* colour than you are for *ball*₁.) In particular, the critic may claim that one could not buy the premise unless you already believed the principle of indifference and its concomitant claim that the greater the number of possible outcomes, the thinner your confidence should be when it is spread over those outcomes. The critic might support their objection by pointing out that a subjectivist or an advocate of the imprecise view could not accept the key intuition of the paper while retaining their earlier views. For such reasons, they may think this paper's argument lacks dialectical value—it's useless in the debate about the views it supposedly bears on.

In reply, I think that the argument has two salient kinds of dialectical value. The first is for the agnostics—so to speak. We could suppose that someone does not already accept either the principle of indifference or the wide interval view. In this sense, they are agnostic between them, but they then find that the argument's key premise qua intuition is a compelling reason in favour of the principle. Here, the acceptance of the premise first *precedes* and then *leads* to the acceptance of the principle. And for the record, this was the position I was in when I was initially torn between the two competing views. The second kind of value concerns non-agnostics who favour the wide interval view. In this case, the idea is that their commitment to the wide interval view may be overpowered by their intuitions about a thought experiment they had not considered before—namely, the two-urn thought experiment in Sect. 2. For example, we could imagine that such a person buys the wide interval view because of the problems in finding a uniquely correct partition (see Sect. 3.8). However, we could also imagine that once they consider the argument, their allegiance changes. Of course, it is still true that they cannot accept the intuition while maintaining their earlier views, but that is the case with many thought experiments. For example, one could not accept the key intuition of Searle's (1980) Chinese room argument while accepting the view of strong AI which he was arguing against.

So I believe this argument can have value in either of these ways without begging the question, and it already has had such value—well, in my case at least.

So those were six objections to some premises of this paper's argument. What follows are three objections to the argument's significance.¹⁹

3.7 Stochastic Ignorance is Unrealistic

One objection is that the argument for the principle of indifference is insignificant because situations of stochastic ignorance seldom, if ever, arise in order for the principle of indifference to even apply to them.

While it is obvious that we are often not totally stochastically ignorant, this does not mean that it is obvious that stochastic ignorance is unrealistic.

To respond to this objection, one would ideally list numerous realistic cases involving total stochastic ignorance, but for brevity's sake, I content myself with one. Suppose Josh is considering buying flowers that are on special for his girlfriend Nancy. He faces a choice between purple or red hyacinths, but he cannot remember whether Nancy's favourite colour was purple or, alternatively, whether it was red. His memory does not suggest that she

¹⁹ The reader may notice that these next two objections are general objections to the principle of indifference rather than specifically to this paper's argument itself. Nevertheless, I discuss these objections here since a few commentators on the argument have raised them, essentially claiming that the argument is insignificant if it is ultimately for a principle that could not plausibly address these obvious challenges.

has one colour preference over another, nor does he have evidence about the frequency with which people of any sort prefer purple over red. In cases like these, Josh is plausibly totally stochastically ignorant and the principle of indifference prescribes an arguably rational probability assignment of $\frac{1}{2}$ that Nancy will prefer the purple flowers.²⁰ This is an example that, while hypothetical, is not too unrealistic and thus suggests that situations of total stochastic ignorance can frequently arise. I suspect that the reader could probably think of other realistic situations of total stochastic ignorance too.

3.8 Uniquely Correct Partitions are Unrealistic or Problematic

The critic may alternatively attack the significance of the argument by instead asserting that cases of uniquely correct partitions seldom arise in order for the principle of indifference to apply to them.

This is especially the case since one can occasionally construct alternative partitions where the principle of indifference could yield conflicting results. This fact was noted by early proponents of the principle, including Keynes (2010 [1921], Ch. 4), and it has received extensive discussion in recent years (Bangu 2009; White 2010; Decker 2012; Smith 2015; Rinard 2014). To give one kind of example, suppose that a prize is behind one of three doors that are in front of you, each uniquely numbered from 1 to 3. One partition of the possibilities is {the prize is behind door 1, the prize is behind door 2, the prize is behind door 3}, but another partition of the possibilities is {the prize is behind door 1, the prize is behind door x } where the prize is behind door x iff it is behind door 2 or door 3. An indifferent probability distribution over the former partition would assign a probability of $\frac{1}{3}$ to the proposition that door 1 conceals the prize; in contrast, an indifferent probability distribution over the latter partition would assign a conflicting probability of $\frac{1}{2}$ to the proposition that door 1 conceals the prize.

To assess this objection, it might be useful to have a conceptual analysis of what a uniquely correct partition is.

But while there are candidates for what might make a partition uniquely correct, I think it is neither feasible nor necessary to outline and defend any such candidate here.²¹ Such a topic may require a paper length treatment by itself—not a mere subsection’s worth like this. And even though it may be useful, we do not need to give an analysis of the concept of a uniquely correct partition in order to understand it. While a ‘uniquely correct partition’ may not be an undefinable or primitive expression like, say, purpleness might be, we can

²⁰ A critic may attempt to level an objection against this example on the basis of difficulties in characterising and partitioning colour predicates. Nevertheless, we can suppose realistically that in this circumstance, the flowers obviously fall under exactly one of the colour categories such that Nancy would prefer one set of flowers over the other, but Josh lacks evidence to discern what her preference would be.

²¹ For example, here’s one candidate: a partition of n propositions $\{p_1, \dots, p_n\}$ is uniquely correct iff (1) the most fine grained a posteriori knowledge or evidence one has about the truth of any proposition in $\{p_1, \dots, p_n\}$ is that exactly one of the n propositions is true and (2) all other a posteriori knowledge about the truth of any such proposition is implied by this fine grained knowledge. This is similar to Keynes’s proposal that the elements of the partition be “indivisible” in the sense that they cannot be “further split up” into alternative elements (Keynes 2010 [1921], 67–68); this much is captured with the clause that the partition be fine-grained. However, the clause differs in that it accords a privileged role to a posteriori knowledge of the possible outcomes rather than regarding it on par with a priori knowledge of possible outcomes. In any case, I am sure this candidate is not the final word and is probably problematic, although a whole paper could be devoted to exploring whether any other alternative fares better.

grasp the concept of a uniquely correct partition without an explication of exactly what it is, somewhat similarly to how we can grasp the concept of purpleness without an explication of what it is. I think most (if not all) philosophers would at least grasp what is being meant when someone says that a uniquely correct partition is {the prize is behind door 1, the prize is behind door 2, the prize is behind door 3} rather than {the prize is behind door 1, the prize is behind door x }.

The important question, however, is whether uniquely correct partitions are ever present in situations of epistemological interest. And I think they are. Perhaps there are no uniquely correct partitions in problematic cases like those of so-called *Bertrand-style paradoxes* (White 2010), but not all cases of total stochastic ignorance are so problematic. The case of the urns and Josh's colour dilemma are arguably somewhat realistic examples of uniquely correct partitions. Inasmuch as the principle of indifference is supported by the intuition that one's confidence in any possible outcome should decrease with the addition of outcomes to the space of possible outcomes, so too may this same support accrue to the notion that there are uniquely correct partitions. So we can think of this paper's argument as not just being an argument for the principle of indifference, but as involving an example of, and argument for, cases where there are uniquely correct partitions.

Now others may be more pessimistic about the possibility of uniquely correct partitions. In particular, Meacham has argued that worries about language dependence "arise for every case of indifference" (Meacham 2014, 1196), thus threatening the possibility that there never is a uniquely correct partition. For example, he discusses the aforementioned case of the three doors. Instead of going indifferent, so to speak, over the partition of the number of doors, he suggests that we could go indifferent over a much less obvious partition involving inverse distance along a spatial dimension.

I am not persuaded by this line of reasoning. It argues against the principle of indifference by introducing further information that creates a partition that in some sense competes with the legitimacy of the more obvious partition of outcomes; but such information is not present in all putative cases of uniquely correct partitions.²² Consider, for example, the case of the urns above or the case where Josh is considering Nancy's favourite colour. It is dubious that there is information in these cases which could furnish a competing partition (or, at the very least, we could realistically suppose that there is no such information).

Sure, one can accept that there may be relevant partitions which would mean that there would be no uniquely correct partition *if one did know of them*. But as long as one is unaware of such partitions, one can suitably apply the principle of indifference. To some, however, it may seem odd that the applicability of the principle of indifference depends on ignorance about such partitions. However, it should be no less odd than the presumably popular intuition that Jenny should have a credence of 0.99 that the Mexican has brown eyes, but only if she is ignorant about the Mexican belonging to the entirely blue-eyed Jenkins family.

²² As mentioned, Meacham also appeals to the concepts of inverse time and inverse distance to argue against the principle of indifference and its application in two particular cases. Unfortunately, I am not knowledgeable about the relevant physics. However, I have discussed Meacham's paper with Brendon Brewer, a statistician and physicist, and he informs me that most, if not all, physicists would probably find Meacham's utilisation of the concepts quite unintuitive. This is because they would be inclined to give ontological priority to more familiar concepts of time and space which accord with the way that physicists think of physical symmetry. Nonetheless, I think that the above response to Meacham's worries suffices irrespective of the physics; and presumably an in-depth analysis of his appeal to physics would take me well out of the domain of formal epistemology.

So given that many cases could realistically lack competing partitions of which the agent is aware, it is arguable that there are often cases of uniquely correct partitions despite Meacham's worries relating to inverse distance, inverse time and the like.

The critic might further object that we should not believe in such partitions unless we can specify why we are justified in taking some to be correct and others to be incorrect. Yet this is dubious for me and for certain others since, for us, intuition intuitively suffices to sift the correct partitions from the incorrect, at least in many cases like those of Josh's colour dilemma and the urn. Indeed, this is somewhat analogous to how we intuitively know the extensions of many concepts without readily having conceptual analyses of such concepts, even though there may be some vagueness in those concepts. I have never seen, for example, a thorough analysis of precisely what it takes for an object to be a chair or for a person to qualify as a Maori person, even though the extension of these concepts is somewhat unclear. (Does a chair need to have exactly four or more legs or could a "chair" have one leg or even be legless if it hangs from a ceiling? And does a Maori person need have at least a certain percentage of Maori blood or cultural values?) The point is that we nevertheless can have an intuitive understanding of what objects are chairs and what people are Maori, one that is adequate for many cases and may lack explicable justifications. Likewise, we can have an intuitive understanding of uniquely correct partitions that is similarly adequate, at least for many cases.

The principle of indifference, then, may rely on an intuitive understanding of such partitions for its application, but this does not entail that it is incomplete or problematic any more than many other principles which likewise rely on intuitive understandings of concepts for their application.

Consequently, I leave the important subject of uniquely correct partitions as one for future research.

3.9 Frequency-Credence Principles and the Principle of Indifference

Another objection is that this paper's argument has not established anything controversial at all. Rather the critic may assert that the principle of indifference follows from an uncontroversial principle called *frequency-credence* and that the argument merely restates this fact.

Here is a rough version of a frequency-credence principle, loosely following White's (2010) articulation of it. Let a be a constant denoting some entity (such as a proposition or a concretely-existing object), let F and G be one-place predicates and let sets of F s and G s be sets of entities satisfying the formulas Fx and Gx respectively. Then, informally, White's frequency-credence principle is essentially that:

If (i) you know that a is an F , (ii) you know that the proportion of F s that are G s is x and (iii) you have no other evidence bearing on whether a is a G , then your credence that a is a G should be x .

To illustrate the principle, let a stand for $ball_j$, Fx stand for " x is a ball from urn_j " and Gx stand for " x is black". Then, if (i) you know that $ball_j$ is a ball from urn_j , (ii) you know that the proportion of balls from urn_j that are black is, say, $\frac{9}{10}$ and (iii) you have no other evidence bearing on whether $ball_j$ is black, then your credence that $ball_j$ is black should be $\frac{9}{10}$. This result is intuitive, and Kaplan and others would endorse something like this principle.

However, according to White's "argument from statistical inference", the frequency-credence principle implies the principle of indifference. For example, suppose you know nothing of the ratio of coloured balls in urn_j . Nevertheless, you know something about the partition $\{ball_j \text{ is black, } ball_j \text{ is white}\}$, namely, you know that exactly one member of the partition is true. Therefore, (i) you know that the proposition that $ball_j$ is black is a member of the partition, (ii) you know that the proportion of members of the partition that are true is $\frac{1}{2}$ and (iii) you have no other evidence bearing on whether the proposition that $ball_j$ is black is true. Therefore, White would claim that, by the frequency-credence principle and conditions (i)–(iii), you should have a credence of $\frac{1}{2}$ that $ball_j$ is black, essentially amounting to a validation of the principle of indifference. White's argument from statistical inference, then, is the fourth argument of special relevance to this paper that I alluded to in the introduction.

The problem with this objection is that it is dubious that many philosophers would accept the frequency-credence principle as White construes it. The reason for this is that condition (ii) conflates two distinct kinds of proportion. Suppose you now know both that the proportion of balls from urn_j that are black is $\frac{9}{10}$ and that the proportion of propositions in the aforementioned partition that are true is $\frac{1}{2}$. One proportion is a proportion of *concretely existing objects that have a property*; the other proportion is a proportion of *abstract propositions that are true*. Call the former kind of proportion a *concrete proportion* and the latter kind an *abstract proportion*.

Clearly there is a significant epistemic difference between having knowledge of a concrete proportion and knowledge of an abstract proportion. For example, you know about both the concrete and the abstract proportions above, but presumably your credence that the ball is black should equal the concrete proportion of $\frac{9}{10}$ instead of the abstract proportion of $\frac{1}{2}$. After all, even though the ball is either black or white, you know that 9 out of 10 of the balls from urn_j are black and that $ball_j$ is one of those balls; so you should have a credence of $\frac{9}{10}$ that it is black (assuming that you have no other relevant evidence). For some reason, then, knowledge of the concrete proportion has a kind of epistemic authority over what your credence should be that knowledge of the abstract proportion does not.

On the basis of this difference, the opponent of the principle of indifference may coherently reject the principle of indifference which concerns abstract proportions while accepting a frequency-credence principle that concerns concrete proportions:

FC*: If (i) you know that a is an F , (ii) you know that the proportion of F s that are G s is x where this proportion is a concrete proportion and (iii) you have no other evidence bearing on whether a is a G , then your credence that a is a G should be x .

Perhaps White sought to discredit the wide interval view with his coin puzzle precisely because FC* by itself is not sufficient to vindicate the principle of indifference.

Note, however, that even though the opponents of the principle of indifference may accept FC*, this paper's argument provides some motivation *independently of FC** to accept the principle of indifference.

Hence, the objection that the argument is insignificant because the principle of indifference is entailed by an uncontroversial frequency-credence principle is dubious precisely because the frequency-credence principle that is uncontroversial is one which presumably concerns concrete proportions and which does not entail the principle of indifference.²³

²³ Thorn (2017) is an example of someone who rejects the principle of indifference which concerns abstract proportions while accepting a frequency-credence principle that concerns concrete proportions.

4 Conclusion and Future Research

I have presented this paper's argument for the principle of indifference and have defended it from several objections. The argument involved a thought experiment which invokes the intuition that in cases of total stochastic ignorance about a uniquely correct partition of possible outcomes, one's confidence in any particular outcome being true should decrease with the addition of outcomes to the space of possible outcomes. This intuition is taken to support the principle of indifference for three reasons. One of these is that the principle of indifference is the end product of distilling our intuitions about the thought experiment from irrelevant details, taking into account only the features that matter for what our credences should be. Relatedly, the second reason is that the requirement to be coherent across our probability assignments favours accepting the principle of indifference as a general guide to inferences about the cases which it concerns. A third reason is that, arguably, the principle of indifference can explain the intuitions about the thought experiment.

The argument may not necessarily be a decisive adjudicator between our two contenders—the wide interval view and principle of indifference—but I believe it is a significant consideration nevertheless.

In any case, this paper has touched on topics that might be fruitful areas for future research. One concerns how to give a conceptual analysis of objective chances, stochastic ignorance and related concepts. Another topic concerns how to give a conceptual analysis of uniquely correct partitions and to explicate what it is that makes partitions uniquely correct. As I have argued, the fruitful application of the principle of indifference relies on an intuitive—albeit adequate—understanding of such concepts, similarly to many other principles and concepts. Regardless, analyses and explications of such concepts may delve deeper into an interesting topic—that is, the question of precisely which circumstances the principle of indifference might apply to and why it might do so.

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